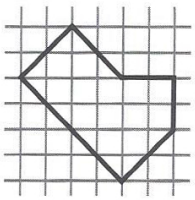
Problem set 5, assigned on 10/16/22

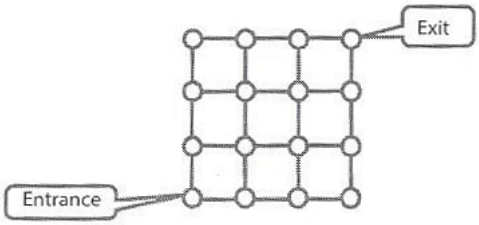
1. Cut the square in the picture into three equal parts. (You can cut along the grid lines and diagonals of small squares.)



2. Eight knights took part in a 3-contest tournament. They competed in archery, sword fighting, and lance throwing. For each contest, a knight was awarded 0, 1, or 2 points. Prove that at least 2 of these knights earned the same total number of points.

3. With a red marker, Margareta marked 3 points with integer coordinates on a number line. With a blue marker, Angelina marked a midpoint for every pair of red points. Prove that at least one of the blue points has an integer coordinate.

4. The halls of the Haunted Labyrinth can be walked to the right and upward only. Each room of the Labyrinth (marked by a circle) is inhabited by a magical creature - an elf, a leprechaun, or a fairy. If you pass through a room where an elf lives, he'll give you 3 coins; if you pass through a fairy's room, you'll get 6 coins; and a leprechaun will give you nothing. Twelve kids enter the labyrinth. Prove that at least 2 will exit on the other side with the same number of coins.



5. (This problem may be a bit more challenging...) The 6th-grade math circle student Emilio wrote a computer program for his house robot, Basil. Starting from 1, Basil should keep writing bigger and bigger numbers formed by 1's: 1, 11, 111, etc. The program terminates when Basil writes a number that is a multiple of 19. Prove that the program will terminate in fewer than 20 steps. [Hint: Think about the Pigeonhole Principle, and about the remainders when you divide these numbers by 19.]