

Problem set 3, assigned on 10/2/22

- 1.** A broken calculator can only do several operations: multiply by 2, divide by 2, multiply by 3, divide by 3, multiply by 5, and divide by 5. Using this calculator any number of times, could you start with the number 12 and end up with 49?
- 2.** The numbers 1 through 12 are written on a board. You can erase any 2 of these numbers (call them a and b) and replace them with the number $a + b - 1$. After 11 such operations, there will be just 1 number left. What could this number be?
- 3.** There are 6 trees growing along Park Lane, and a bird is sitting in each of them. Every time a car passes, exactly 2 birds fly up and go to the next tree in either direction. Can all birds gather in the same tree?
- 4.** If a magician puts 1 dove into his hat, he pulls out 2 rabbits and 2 flowers from it. If the magician puts 1 rabbit in he pulls out 2 flowers and 2 doves. If he puts 1 flower in, he pulls out 1 rabbit and 3 doves. The magician starts with 1 rabbit. Could he end up with the same number of rabbits, doves, and flowers after performing his hat trick several times?
- 5.** Two identical cups, one filled with coffee and another with milk, have been placed on the table. You take a spoonful of milk from the milk cup and put into the coffee cup. Next, You take a spoonful of the mixture from the coffee cup and put it into the milk cup. Do you have more milk in the coffee cup now or more coffee in the milk cup?