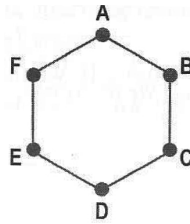


Problem set 8, assigned on 11/17/19

1. Connect 6 points with curved lines such that the lines would not intersect and each point would have 4 lines ending in it.

2. The 6 towns in the country of Soupstop are connected by 6 roads as shown. Several flying saucers landed on each of the roads. The captain of each saucer sent an alien to each of the 2 nearest towns to steal a fresh bowl of soup (everyone knows that flying saucers use chicken soup as their fuel). As a result,  $a$  bowls of soup were stolen in  $A$ ,  $b$  bowls went missing in  $B$ ,  $C$  lost  $c$  bowls,  $D$  lost  $d$  bowls, and in  $E$  the aliens snatched  $e$  bowls of soup. How many bowls of chicken soup were stolen in  $F$ ?



3. In a graph of 40 vertices, the degree of each vertex is at least 20. Prove that the graph is connected.

4. The Knights and Liars Island has two types of people: knights and liars. Knights always say the truth, and liars always lie. Every person from the Knights and Liars Island said 2 phrases:

- "I have an odd number of knight friends."
- "I have an even number of liar friends."

Is the total number of Islanders odd or even?

5. The remote country of Booroodoo has 2 airlines, Royal and Republican. Each Booroodoo airport is served by 4 flights, 2 of which are Royal and 2 are Republican. Out of all these flights, only 2 are international. Prove that these 2 flights belong to the same airline.