Problem set 9, assigned on 11/24/19

1. Can you come up with 99 positive integer numbers such that their sum and their product are both equal to 99?

2.
(a) Can you draw a graph with vertices of degrees 8, 6, 5, 4, 4, 3, 2, 2?
(b) Can you draw a graph with vertices of degrees 7, 7, 6, 5, 4, 2, 2, 1?
(c) Can you draw a graph with vertices of degrees 6, 6, 6, 5, 5, 3, 2. 2?
For each part either show how or explain why this is not possible.

3. In a graph, every vertex is either blue or green. Every green vertex is connected by an edge with 9 blue and 6 green vertices, and every blue vertex is connected by an edge with 5 blue and 10 green vertices. Does this graph have more green or more blue vertices?

4. In the network of 101 computers, each one is connected with exactly k computers. What are the possible values of k?

5. A ship hold is a rectangular-shaped room. Through the hole in the hull the water started to leak into the hold. The water pump, which was immediately turned on, was not powerful enough. The water level was rising steadily; it reached the height of 20 cm in 10 minutes. At this moment the second pump, identical to the first one, was turned on. In 10 minutes, the water level went down to 10 cm. At the same moment the leak was fixed. How long will it take for the two pumps to pump all the water out?

6. Captain Flint has 60 pirates in his crew. Each pirate dislikes exactly one other pirate. Prove that the captain can split the pirates into 3 groups such that within each group no one dislikes anyone else. ("Dislike" is not necessarily symmetrical. If Pirate A dislikes Pirate B, that does not mean that Pirate B dislikes Pirate A.)