

Problem set 7, assigned on 11/3/19

1. Our office has 9 computers. My manager wants to link them into a network so that every computer would be connected with exactly 4 computers by network cables.
 - (a) How many cables will we need? Can you draw such a configuration?
 - (b) The manager decided to connect every computer with exactly 5 other computers. Is this a possible configuration?
2. Martian amoebas gemmate (they multiply by division). When a red amoeba divides; it splits itself into 5 blue amoebas; When a blue amoeba divides, it splits into 7 red amoebas. When my spaceship left Mars I put 1 amoeba into a jar. Upon arriving on Earth, the customs officer found 100 amoebas in that jar. Why does he keep searching my ship insisting that some amoebas must have escaped?
3. The 99 greatest scientists of Mars and Venus are seated evenly around a circular table. If any scientist sees 2 colleagues from her own planet sitting an equal number of seats to her left and right she waves to them. For example, if you are from Mars and the scientists sitting 2 seats to your left and right are also from Mars, you will wave to them. Prove that at least 1 of the 99 scientists will be waving; no matter how they are seated around the table.
4. Can you find a map on the torus which needs **six** colors (so that any two countries that share a border are colored differently)?