

Problem set 5, assigned on 10/20/19

1. Convert the following numbers from binary (base 2) to decimal (base 10):

- (a) 111
- (b) 1101
- (c) 10000000.

2. Convert the following numbers from decimal to binary:

- (a) 8
- (b) 15
- (c) 32
- (d) 31
- (e) 53.

3. Add these binary numbers using the column addition algorithm:

- (a) $1000+101$
- (b) $11011+111111$

4. In class we discussed the game of Nim with heap sizes 3, 4, 5, and a winning strategy for the first player using the notion of *nim-sum*. Which player has a winning strategy when heap sizes are 3, 4, 6? How about 3, 4, 7?

5. Recall the operations S (swap) and R (rotate) on rational tangles, discussed in class: in terms of the corresponding *tangle number*, S corresponds to adding 1, and R gives a negative reciprocal: it replaces x with $-\frac{1}{x}$.

The following instructions show how to untangle any rational tangle:

- If the tangle number is negative then to SWAP and add 1 to the tangle number to obtain a new number.
- If the tangle number is positive, then ROTATE and take the negative reciprocal of the tangle number.
- If the tangle number is zero, STOP. The ropes are untangled.

Using these two operations, can you untangle the rational tangle corresponding to the number 4?