

## Problem set 9 (Assigned on November 7th)

Remember many cartoon characters have four fingers on one hand, and therefore it is probable that they would use only eight digits for counting:  $0, 1, 2, \dots, 7$ . For example, number 18 does not exist in the cartoon world. Since the number  $24_{(8)}$  denotes two full hands and 4 fingers, in 8-fingered world, it describes  $2 \times 8 + 4 = 20$  objects. The subscript (8) means that we are in 8-fingered world. Simpsons are living in one of such worlds, and Bart and Lisa have several homework problems. Can you help them?



1. Lisa Simpson (8 fingers) in the second grade learned how to add two- and three-digit numbers without carrying. Can you help her with the homework:

$$\begin{array}{r} 11_{(8)} \\ + 23_{(8)} \\ \hline \end{array} \qquad \begin{array}{r} 125_{(8)} \\ + 230_{(8)} \\ \hline \end{array}$$

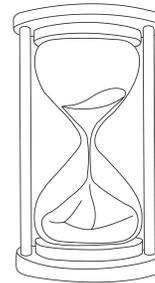
2. Bart in the fourth grade has harder problems:

$$\begin{array}{r} 11_{(8)} \\ + 27_{(8)} \\ \hline \end{array} \qquad \begin{array}{r} 72_{(8)} \\ + 7_{(8)} \\ \hline \end{array}$$

3. (*Repetition from the last homework*) (a) If you have a 7-minute hourglass and 11-minute hourglass, can you use them to boil an egg for 15 minutes? (b) Is it possible to start the egg boiling at the same time you start an hourglass and still boil the egg for 15 minutes?

Note that hourglass is a device that can measure precisely the given time interval, but not any shorter one.

*Discussion.* In the class, we discovered a solution to problem (a): Run both hourglasses at the beginning and wait till 7-minute one stops. At that moment, there are still 4 minutes left in 11-minute hourglass. Use those combined with 11 minute hourglass to measure 15 minutes. For part (b), we again run both hourglasses at the time we start boiling an egg. After 7 minutes, we have 4 minutes left on 11 minute hourglass. Use these 4 minutes twice (this is tricky!) to obtain  $7 + 4 + 4 = 15$  minutes.



4. Numbers 1, 2, 3 are placed at the vertices of the triangle. Arrange numbers 4, 5, 6, 7, 8, and 9 along the sides of the triangle (maybe more than one number per side) such that numbers along each side add to 17.

