

**CORRIGENDUM TO “NOTES ON INTERPOLATION IN THE
 GENERALIZED SCHUR CLASS. II. NUDEL’MAN’S PROBLEM”**

D. ALPAY, A. DIJKSMA, AND J. ROVNYAK

ABSTRACT. This note identifies errors in the joint paper [Trans. Amer. Math. Soc. **355** (2003), 813–836] by the authors and T. Constantinescu. The Main Theorem requires a stronger hypothesis. The applications to interpolation problems are affected by this change; some survive, but others do not and are withdrawn.

The Main Theorem in [1, p. 816] has gaps. In the proof of Part (1), the statement on p. 832, line 6, that the operator $X_0 = Y_0 + K$ is bounded is an error, because no reason is given why the finite-rank summand K is bounded. A similar error occurs in Part (2) on p. 833, line 15, where again it is asserted without justification that X_0 is bounded. A correct version of the Main Theorem is obtained by replacing the condition (iii) in Definition 2.1 with a stronger version:

(iii') there is an $M > 0$ such that $\sum_{j=0}^{\infty} |(A^j b, x')|^2 \leq M \sum_{j=0}^{\infty} |(A^j c, x')|^2$ for all x' in \mathcal{D} .

Condition (iii') makes X_0 bounded from the start, and then the proof of the Main Theorem goes through as written. The Main Theorem (Alternative Form) on p. 834 is correct as written provided that condition (iii') is adopted.

A number of applications survive this change. Theorem 3.1 on Pick-Nevalinna interpolation is true as stated. In the proof on p. 817, it is only necessary to add an argument that (iii') is satisfied. In the same notation, for any x in \mathbb{C}^n ,

$$\begin{aligned} \sum_{j=0}^{\infty} (A^j b, x) z^j &= \sum_{j=0}^{\infty} \sum_{k=1}^n z_k^j w_k x_k z^j = \sum_{k=1}^n \frac{w_k x_k}{1 - z_k z}, \\ \sum_{j=0}^{\infty} (A^j c, x) z^j &= \sum_{j=0}^{\infty} \sum_{k=1}^n z_k^j x_k z^j = \sum_{k=1}^n \frac{x_k}{1 - z_k z}. \end{aligned}$$

Thus (iii') requires that for some $M > 0$,

$$(1) \quad \left\| \sum_{k=1}^n \frac{w_k x_k}{1 - z_k z} \right\|^2 \leq M \left\| \sum_{k=1}^n \frac{x_k}{1 - z_k z} \right\|^2, \quad x \in \mathbb{C}^n,$$

2010 *Mathematics Subject Classification*. Primary 47A57; Secondary 30E05, 47B32, 47B50, 42A50.

Key words and phrases. Interpolation, negative squares, Nudel'man, Nevanlinna, Pick, Carathéodory, Fejér, generalized Schur class, generalized Nevanlinna class, commutant lifting.

where $\|\cdot\|$ is the norm in H^2 . The inequality (1) is easily brought to the form of a matrix inequality $WCW^* \leq MC$, where

$$C = \left[\frac{1}{1 - z_i \bar{z}_j} \right]_{i,j=1}^n, \quad W = \text{diag} \{w_1, \dots, w_n\}.$$

Here $C = [\langle g_i, g_j \rangle]_{i,j=1}^n$, where $g_k(z) = 1/(1 - z_k z)$, $k = 1, \dots, n$. Since z_1, \dots, z_n are distinct, the functions g_1, \dots, g_n are linearly independent. Therefore C is non-negative and invertible [2, p. 407]. Thus $\delta I_n \leq C \leq \mu I_n$ for some $\delta, \mu > 0$. If $\eta = \max\{|w_k|: k = 1, \dots, n\}$, then

$$WCW^* \leq \mu \eta^2 I_n \leq \delta^{-1} \mu \eta^2 C,$$

which implies (iii') with $M = \delta^{-1} \mu \eta^2$.

Theorem 3.4 and Corollary 3.5 on Carathéodory-Fejér interpolation are true as stated. To verify (iii') in this example, we proceed as in the proof of Theorem 3.4 in [1, p. 818]. The sums in (iii') are finite because $A^k = 0$ for all $k > n$. A short calculation shows that for all x in \mathbb{C}^{n+1} ,

$$\sum_{j=0}^n |(A^j b, x)|^2 = \|Tx\|^2,$$

where $\|\cdot\|$ is the Euclidean norm on \mathbb{C}^{n+1} and

$$T = \begin{bmatrix} w_0 & w_1 & \cdots & w_n \\ 0 & w_0 & \cdots & w_{n-1} \\ & & \cdots & \\ 0 & 0 & \cdots & w_0 \end{bmatrix}.$$

Since also

$$\sum_{j=0}^n |(A^j c, x)|^2 = \|x\|^2,$$

condition (iii') simply asserts that T is bounded as an operator on \mathbb{C}^{n+1} . This is clearly true. Thus Theorem 3.4 follows from the corrected form of the Main Theorem.

Theorem 3.6 on Sarason generalized interpolation is true as stated. In the proof, (iii') follows from the displayed formulas in lines 3 and 5 on p. 820.

Other applications do not survive because (iii') is not satisfied or cannot readily be verified. Theorem 3.2, its corollary, and Theorem 3.7 are in this category and are withdrawn. The boundary results in Theorem 3.8, its corollary, and the results in Section 4 are withdrawn for the same reason. An exception here is the Alternative form of Corollary 3.9 on p. 824, which does not use the Main Theorem and is correct as written. The Example on p. 825 remains valid when (iii) is replaced by (iii'). Partial results suggest that the withdrawn boundary theorems now become reasonable open problems. The authors plan to discuss these matters in a future work.

An anonymous reviewer provided ideas by which Theorem 3.2 might be saved. We do not know if they work and leave the validity of Theorem 3.2 as an open problem.

REFERENCES

- [1] D. Alpay, T. Constantinescu, A. Dijksma, and J. Rovnyak, *Notes on interpolation in the generalized Schur class. II. Nudelman’s problem*, Trans. Amer. Math. Soc. **355** (2003), no. 2, 813–836. MR 1932727
- [2] R. A. Horn and C. R. Johnson, *Matrix analysis*, Cambridge University Press, Cambridge, 1990, Corrected reprint of the 1985 original. MR 1084815

FOSTER G. AND MARY MCGAW PROFESSORSHIP IN MATHEMATICAL SCIENCES, FACULTY OF MATHEMATICS, PHYSICS, AND COMPUTATION, SCHMID COLLEGE OF SCIENCE AND TECHNOLOGY, CHAPMAN UNIVERSITY, ONE UNIVERSITY DRIVE, ORANGE, CA 92866, USA

E-mail address: `alpay@chapman.edu`

JOHANN BERNOULLI INSTITUTE OF MATHEMATICS AND COMPUTER SCIENCE, UNIVERSITY OF GRONINGEN, P. O. BOX 407, 9700 AK GRONINGEN, THE NETHERLANDS

E-mail address: `a.dijksma@rug.nl`

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF VIRGINIA, P.O. BOX 400137, CHARLOTTESVILLE, VA 22904, USA

E-mail address: `rovnyak@virginia.edu`