

Selection, Optimization, Compensation, and Equilibrium Dynamics

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Abstract

One of the major theoretic frameworks through which human development is studied is a process-oriented model involving selection, optimization, and compensation. These three processes each provide accounts for methods by which gains are maximized and losses minimized throughout the lifespan, and in particular during later life. These processes can be cast within the framework of dynamical systems theory and then modeled using differential equations. The current article will review basic tenets of selection, optimization, and compensation whilst introducing language and concepts from dynamical systems. Four categories of interindividual differences and intraindividual variability in dynamics are then described and discussed in the context of selection, optimization, and compensation.

Keywords: Selection, Optimization, Compensation; Dynamical Systems Analysis; Lifespan Development

Development over the lifespan involves both processes that operate more-or-less outside of conscious awareness as well as conscious choices of behaviors. Both types of processes can be viewed within the framework of regulation of behavior with respect to goals. P. B. Baltes and Baltes (1990) proposed a meta-theory for three adaptive regulatory processes operating during the life course: selection, optimization, and compensation. These processes are advanced as helping account for successful development and aging from an intraindividual process perspective — How an individual develops is deemed to be a continual systematic interplay between current internal states and capacities of the individual and the environmental demands and contextual opportunities in which the individual is immersed.

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Process oriented theories as underpinnings for developmental adaptation and change have had increasing influence on developmental research (e.g. Burke & Shafto, 2008; Finucane, Mertz, Slovic, & Schmidt, 2005; Kramer, Bherer, Colcombe, Dong, & Greenough, 2004). The current article will discuss developmental processes within the framework of dynamical systems analysis and aims to weave the ideas of equilibrium dynamics into the fabric of selection, optimization, and compensation.

Selection, Optimization, and Compensation

The meta-theory of selection, optimization, and compensation (P. B. Baltes, Staudinger, & Lindenberger, 1999) has been elaborated over the past two decades from many perspectives and within many contexts (M. M. Baltes & Carstensen, 1996; P. B. Baltes, 1997). Examples include conscious, goal-oriented adaptation to aging measured through self-report (e.g. Freund & Baltes, 2002), short-term processes such as dual task compensation (e.g. Li, Lindenberger, Freund, & Baltes, 2001), and age-related differences emotion-regulation (e.g. Urry & Gross, 2010). While a full review of selection, optimization, and compensation is outside the scope of this article, a brief overview is presented to introduce to the three individual concepts and some of the possible contextualized interactions between them.

Selection

The mechanism of selection is generally presented as a process of goal choice. There are many possible ways that selection could be represented within the context of an individual's life. For instance selection could be a categorical choice from mutually exclusive alternatives, e.g., "Do I want to go swimming at the beach or camping in the mountains for my holiday?". It could be a selection of the number of members in a set of multiple alternatives, e.g., "How many different hobbies should I pursue?". Or selection could represent a weighting of choices from available alternatives, e.g., "What are the relative importance of work and family as I choose how to spend my time this next year?"

Freund and Baltes (2002) distinguish between *elective selection* as described in the previous paragraph and *loss-based selection* which is a consequence of restricted opportunities and loss of function associated with late life. Taking a lifespan perspective, expanded opportunities and gains in function should also be considered. Thus, the present article will make the distinction *voluntary* and *involuntary* selection. Examples of involuntary selection could be a restriction of choice of daily activities as a consequence of a broken hip, or social isolation due to loss of hearing. Positive examples of voluntary selection could be the additional range of activities available to an infant when she begins to walk, or new travel opportunities afforded to a young adult who learns a second language.

Optimization

Optimization refers to the application of methods to achieve selected goals. In some contexts, this process could involve both acquisition of methods, e.g., learning a new skill, as well as the use of mean in goal-relevant behavior, e.g., applying the skill. The process of optimization of behavior can also be considered within the framework of regulation of behavior. In order to regulate behavior with respect to a goal, an organism must be able

to perceive the mismatch between its current state and the desired state and produce a behavioral change that leads to the desired state.

An obvious example is locomotion. If one is at a workplace and desires to go home, then a serially dependent chain of behaviors can be constructed: in order to arrive at home, one must catch the bus; in order to catch the bus, one must first leave the office building; in order to leave the building, one must first exit one's office; and so on. While many motile behaviors may lead to eventually arriving home, (e.g., driving, taking the bus, walking) some may not (e.g., swimming, skydiving). Optimization involves both the application of the methods relative a particular goal as well as selection of appropriate methods — one must know in which direction to travel (e.g., home is 1 km to the northeast) as well as applying appropriate methods (e.g., walking rather than skydiving).

Optimization may also occur within the context of either non-conscious or non-intentional behaviors. One example is emotion regulation. While emotion regulation may optimize affective states relative to an individual's homeostatic equilibrium, this optimization need not be intentionally invoked, nor even necessarily be within conscious control. This sort of optimization may also be observed in social perception-action contexts such as conversation, mutually directed gaze, or motor tasks.

Compensation

Compensation refers to the use of alternative methods when previously preferred methods become unavailable. Compensation is distinguished from selection in that selection refers to choice of goals rather than choice of methods. In contrast to optimization, compensation is defined by the tradeoff between two or more methods — as one set of methods becomes less useable, another set of methods is substituted. For instance, as age-related presbyopia reduces the ability to focus on nearby objects, an individual may compensate by wearing reading glasses.

Interactions between Selection, Optimization, and Compensation

It should be evident that the three mechanisms of selection, optimization, and compensation are intertwined such that changes in one are almost always going to result in changes in the other two. Some of these changes may be due to conscious use of these strategies as a meta-strategy for successful development and aging. The use of the three strategies appears to have a developmental trajectory in which selection becomes more important during early to middle adulthood, optimization and compensation become more important for the so-called young-old (age 60 to 80) and then the three strategies become more difficult to successfully apply for adults over 80 (Freund & Baltes, 2002). There is also evidence of a positive association between use of the three strategies and higher levels of functioning and successful life management (M. M. Baltes & Lang, 1997; Freund & Baltes, 2002; Li et al., 2001).

The dynamic relationship between the goal selection, optimization of goal achievement by available methods, and compensatory strategies is likely to be complex in several ways. Short term change in each of the three mechanisms may lead to long term change in any or all of the three. However, it is reasonable to expect that these changes may be coupled in systematic relationships that can be studied by longitudinal measurement. With that in mind, we next provide a very short overview of longitudinal measurement and data

analysis and then provide a description of four types of individual developmental differences within the framework of dynamical systems analysis and their relationship to selection, optimization, and compensation.

Longitudinal Measurement

It is important to understand human development as a set of processes that occur within individuals. In order to study the development of individuals, one must design studies with a longitudinal component — repeatedly measuring individuals with targeted intervals of time separating the occasions of measurement. P. B. Baltes and Nesselroade (1979) were influential in defining how the goals of longitudinal research into development should be focused on the processes that lead to development and not simply on differences between age groups. There are many reasons why a sample of individuals at two selected ages may differ: cohort effects; selection due to morbidity or mortality; and selection due to response propensity are but a few of the problems with age-based comparisons. In addition, there can be large individual differences in age at which individuals experience any particular developmental effect attributed to aging. While some of these effects are no doubt due to biological senescence processes, many of the observable behavioral changes associated with age are likely to be the result of processes that operate at time scales much shorter than biological senescence. Short- and long-term processes observed in behavior and aging are likely to be interrelated in a variety of ways (Boker, Molenaar, & Nesselroade, 2009).

Nesselroade (1991) proposed that short-term variability and long-term change, sometimes termed as *trait* and *state* variance, could be considered to be part of a fabric of developmental processes. In his metaphor these sources of variance are the warp and woof of development, emphasizing the long threads and short threads involved in the process of weaving cloth. This view of development has gained considerable traction and led to empirical research in which *measurement bursts* with short (e.g., daily) intervals between observations (Lebo & Nesselroade, 1978) are embedded into a longitudinal design where the measurement bursts are themselves separated by longer (e.g., multi-year) intervals (Ram & Gerstorf, 2009; Ong, Bergeman, & Boker, 2009). This sort of design affords the opportunity to simultaneously model short- and long-term processes, thereby allowing tests of potential relationships between two or more time-scales of measurement.

Dynamical Systems Analysis

Recently, dynamical systems models (Boker & Bisconti, 2006; Hubbard & West, 1991; Kaplan & Glass, 1995; Smith & Thelen, 1993) have begun to be applied to a variety of questions in development and aging including bereavement (Bisconti, Bergeman, & Boker, 2004, 2006), daily stress and affect (Montpetit, Bergeman, Deboeck, Tiberio, & Boker, 2010; Sliwinski, Almeida, Smyth, & Stawski, 2009), cortisol response (Almeida, Piazza, & Stawski, 2009), postural control (Newell, Mayer-Kress, & Liu, 2009; Slobounov, Moss, Slobounov, & Newell, 1998), resiliency (Ong et al., 2009; Sliwinski & Mogle, 2008), and cognition (MacDonald, Li, & Bäckman, 2009; Ram, Rabbitt, Sollery, & Nesselroade, 2005; Sliwinski, Smyth, Hofer, & Stawski, 2006). Dynamical systems analysis is based on the premise that covariance relationships between variables and their rates of change with respect to time can illuminate and simplify the discussion of intraindividual processes.

Short term behavioral processes can be modeled as having dynamics (Carver & Scheier, 2002) that express the change-related relationships between individuals' current states, the states of their environments, and homeostatic equilibria (Boker, 2012). This means of thinking about within-person behavioral variability allows us to consider both stability and change within the same theoretical framework. For instance, an individual may organize her or his behavior in order to maintain stability in the dynamic relationship between goals, environmental demands, and preferred equilibria. An individual's behavior may be organized to help regulate or stabilize this dynamic relationship rather than to obtain stability as defined by unchanging behavior. This type of regulation means that the relationship between the current rate of change in a behavior and the difference between the current state and some desired state or comfort zone may itself be a stable characteristic of an individual. Stable relationships between time derivatives of a system are called *intrinsic dynamics* and can be formally modeled using methods such as difference equations or differential equations.

But just because stable intrinsic dynamics are observed within an individual at one point in time does not imply that all individuals regulate in the same manner or with respect to the same equilibria. Nor does it mean that an individual exhibits no change in his or her regulatory dynamics or maintains an unchanging set of equilibria over time. It is important to distinguish between short-term intrinsic dynamics exhibited at one point in time and longer term changes in equilibria or regulation that can occur as part of the lifespan developmental processes associated with aging. In order to set the stage, we will first introduce what is meant by regulation with respect to an equilibrium and basin of attraction.

Individual Differences in Regulation Relative to Equilibrium

In order to better understand differences in regulation, it is useful to graphically visualize what is often called a *basin of attraction* around an equilibrium. One way to visualize a basin of attraction is to plot the relationship between distance from equilibrium and time derivatives of a system as the slope and/or curvature of a continuous surface as shown in Figure 1. This can give an intuitive sense of how some systems regulate since we can use our intuitive notions of the physical world as a guide. Imagine a marble released near the edge of the bowl in Figure 1-a. The marble would roll down the slope of the bowl and back up the other side, repeating this process until it finally came to rest at the bottom of the bowl. If the bowls sides were steeper, as in Figure 1-b, the marble would roll back and forth more rapidly, producing a regulation with a faster frequency. But again, after some time, the marble would again come to rest exactly in the middle of the bowl. Both Figures 1-a and -b are examples of basins of attraction with *point equilibria*.

Now consider the behavior of a marble dropped at the edge of the basin of attraction shown in Figure 1-c. While the marble would again roll towards the flat bottom of the bowl and likely up the opposite side, we do not know in advance exactly where the marble will end up coming to rest. After some time the marble will stop somewhere on the flat bottom, but this basin of attraction is fundamentally different than the previous two point equilibria in that there is not the same determinacy of final position — this basin of attraction has an *equilibrium set*, sometimes called a “comfort zone” of equally likely final positions for the marble. In such a system, as long as a state variable is within the comfort zone, the system

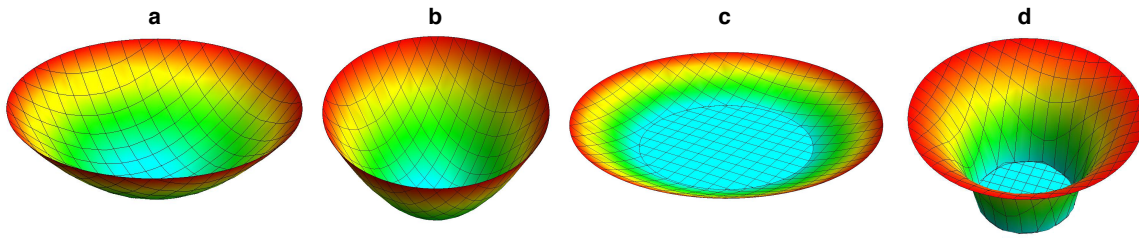


Figure 1. Four basins of attraction characterizing regulation around either a homeostatic point (a and b) or a homeostatic comfort zone (c and d).

does need to engage a regulatory mechanism.

Next, consider the basin of attraction shown in Figure 1–d. In the previous three basins of attraction, the acceleration of the marble is reduced as it nears the equilibrium. However in Figure 1–d, the marble would increase its rate of acceleration as the ball nears the equilibrium, coming to a sudden stop at the bottom. There is a fundamental difference between the shape of Figure 1–d and the other basins in that each of the others has concave curvature whereas Figure 1–d has convex curvature.

In Figure 2 a single basin of attraction morphs from left to right into four separate basins. If a marble were to be released into the single basin of attraction in Figure 2–a, it would roll to the center. Thus, there is a single point–equilibrium in this basin. In Figure 2–b, the single basin becomes wider and flatter as the first stage of morphing into four basins. A marble released into this basin would take longer to come to rest, that is to say regulation with respect to equilibrium is weaker in Figure 2–b than in Figure 2–a. Figure 2–c has four shallow basins, i.e., four weak point equilibria. This is fundamentally different than the single shallow basin. This type of fundamental transition is called a *bifurcation*. A single basin of attraction in Figure 2–b bifurcates into four basins in Figure 2–c. The weaker basins of attraction on either side of this bifurcation is general characteristic of systems: Regulation becomes more weaker near bifurcation, and thus observed trajectories (the path of the marble) is more variable near a bifurcation.

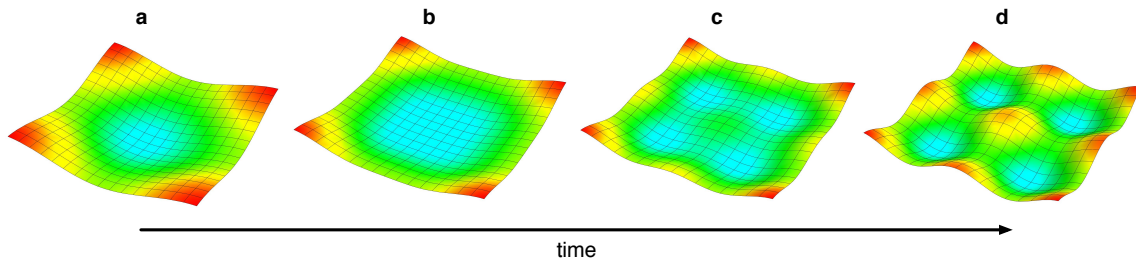


Figure 2. A single basin of attraction, (a and b), morphing into four basins of attraction of increasing strength (c and d). This type of change in an attractor is called bifurcation and could be used as a method of representing changes in choices for selection of goals.

If a marble is released into the attractor surface shown in Figure 2–c, it could be difficult to predict in which of the shallow basins the marble would come to rest. Small unpredictable external influences (in dynamical systems terms, these are called *perturbations*) could result in the marble coming to rest in at any one of the four equilibrium points. Even after the marble was nearly at rest in one basin, small external perturbations could cause the marble to unpredictably switch to another basin. Newly formed equilibria may not appear to be well-regulated with frequent changes between basins of attraction.

In contrast, Figure 2–d has four attractor basins with steeper sides than in Figure 2–c. It would be easier to select in which of the four basins a marble would come to rest. Comparing Figure 2–a and Figure 2–d, it is evident that the bifurcation produced more choices of behavior from which one might select: what was a single point attractor is now four point attractors. In Figure 2–d, simply by pushing the marble in a selected direction, one could move it from one basin to another, i.e., from regulating about one point equilibrium to regulating a different point equilibrium. Thus the process of bifurcation led to increased opportunities for selection.

As suggested by Freund and Baltes (2002), developmental changes in young to middle adulthood may lead to greater number of choices and thus an increasing need for exercising selection as a strategy. This hypothesis of developmental change in attractor complexity can be represented in dynamical systems terms as morphing basins of attraction, as diagrammed in Figure 3. Compared to childhood or late life, young and middle adulthood are represented as having greater complexity of choice due to increased numbers of basins of attraction. Exercising selection as choice of attractor basins would lead to more complex trajectories of behavior in mid-life as opposed to early- or late-life. In this diagram, the late-life basin of attraction is shown as more shallow than the early-life attractor, which would result in trajectories with greater variability but lower complexity in late-life as opposed to mid-life.

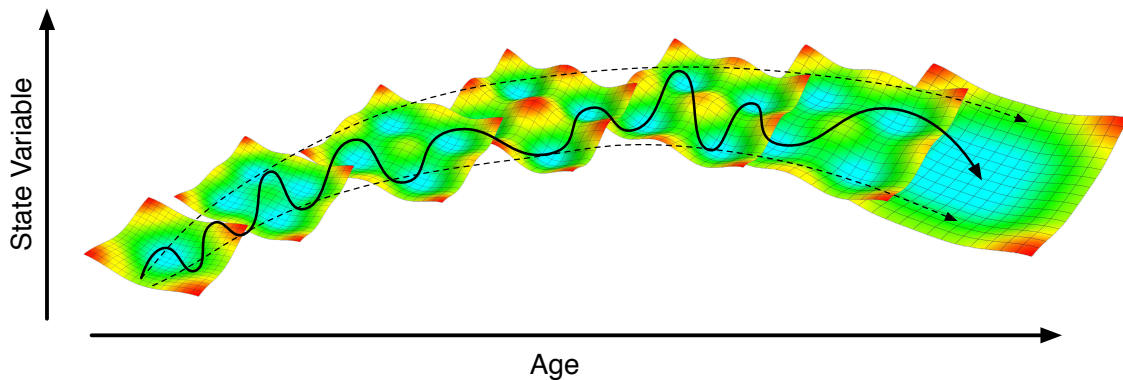


Figure 3. Age-related changes in complexity of selection represented as as developmental change in complexity of basins of attraction. In this hypothetical example, a single basin of attraction in childhood bifurcates into four basins of attraction which then collapse into a single basin of attraction in old age.

If one wishes to understand mechanisms of regulation and how they exhibit devel-

opmental change, then one must focus on the individual in order to extract individual differences in dynamics as well as developmental change in dynamics over time. Thus, although samples from individual behavioral trajectories are the data for analysis, the goal is to describe attractors and attractor changes over time. In other words, the dark fluctuating line in Figure 3 provides the data, but we are not looking to estimate some average trajectory over all individuals. Such an estimation would only produce a single average curve, losing the complexity of individuals' regulatory behaviors. Rather, what we wish to estimate are the attractor basins and how they change over time. In this way we can preserve the potential complexity of ideographic regulation as well as understand nomothetic rules that characterize populations.

There are many ways in which regulatory dynamics could differ between individuals. Similarly, there are a variety of ways in which regulatory dynamics could change as an individual ages. By categorizing individual differences and intraindividual changes in dynamics, we intend to convey some of the richness and specificity that accrue when human behavior and development are considered from a dynamical systems point of view. We will consider four categories in which individuals' dynamics might differ or change: i) individual differences in equilibrium values; ii) intraindividual change in equilibrium values; iii) individual differences in regulation; and iv) intraindividual change in regulation. We then present an instance of selection, optimization and compensation where regulatory dynamics may be stable while the behavioral indicators change over time. Finally, we present a brief discussion of how multiple regulatory processes can be coupled together — selection, optimization and compensation may interact between regulatory processes in complex ways.

Figure 4 presents a summary grid of these sections, organizing the ideas from the next four sections so that specific dynamical systems modeling techniques can be applied. The three columns of the grid are organized around selection, optimization and compensation respectively. The four rows of the grid are organized by the next four sections of this article. Within each cell is a phrase summarizing the concept and a suggestion for a model that might be used to test hypotheses relating to that concept. It is well beyond the scope of this article to delve into specifics about each of these modeling techniques. This organizational grid is presented in order to help guide the interested reader in focusing effort on learning more about methods that may be particularly useful to him or her. References to recommended readings on these methods will be presented in a section near the end of the article.

Individual Differences and Intraindividual Change in Equilibria

The first two categories in which dynamics may differ are individual differences in equilibria and intraindividual change in equilibria. These categories are what is often estimated in developmental studies: differences in individual's means, mean change across time, individual differences in variability, and intraindividual change in variability. From the point of view of dynamical systems, these are only one part of the story, although an important component to the whole picture.

Individual differences in equilibria can be considered from the standpoint of homeostatic set points or comfort zones. One individual may be most comfortable with a moderately high level of a variable, for instance arousal, whereas another individual may be most comfortable in a lower arousal state. Individual differences in equilibria levels can

	Selection	Optimization	Compensation
Individual Differences and Intraindividual Change in Equilibria	<p>Selection of homeostatic equilibria.</p> <p>Differential equations with mixture distribution latent curve means.</p>	<p>Long term developmental dynamics applied to shape and location of short term equilibria.</p> <p>Multilevel differential equations with latent curve means.</p>	<p>Change in shape and location of equilibria in response to external effects.</p> <p>Moderated differential equations with moderated latent curve means.</p>
Individual Differences and Intraindividual Change in Regulation	<p>Selection of regulatory strategies.</p> <p>Regime switching, mixture distribution latent differential equations.</p>	<p>Intrinsic regulation within basins of attraction.</p> <p>Latent differential equations with moderators.</p>	<p>Regulation in response to external effects.</p> <p>Latent differential equations with exogenous variables.</p>
Selection and Adaptation of Behavioral Indicators	<p>Selection of behaviors to regulate and/or markers for latent constructs.</p> <p>Ideographic filter measurement models with constraints on dynamics.</p>	<p>Optimization of measurement model loadings.</p> <p>Time-varying ideographic filters with constraints on dynamics.</p>	<p>Moderation of measurement model loadings by external effects.</p> <p>Moderated ideographic filters with constraints on dynamics.</p>
Coupled Systems	<p>Selection of processes for regulatory coupling.</p> <p>Mixture distribution coupled latent differential equations.</p>	<p>Feedback between regulatory processes and developmental change.</p> <p>Coupled latent differential equations.</p>	<p>Moderation of coupling strength between processes by external effects.</p> <p>Moderated coupled latent differential equations.</p>

Figure 4. Grid relating Selection, Optimization, and Compensation to processes in equilibrium dynamics. Each cell contains a brief example description and possible model types for testing hypotheses.

be modeled as individual differences in the value of the point equilibrium of the associated attractor. Individual differences in values of point equilibria and centroids of equilibrium sets have been observed in a wide range of variables (e.g., Bisconti et al., 2006; Boker & Laurenceau, 2007) and one would be well advised to not rule out individual differences in equilibria values in any variable used as a measure of intraindividual regulation.

Similarly, some individuals may not regulate unless a state variable leaves a relatively large comfort zone of values while other individuals may begin regulating as soon as a state variable deviates from a homeostatic value. While this type of individual difference in equilibrium set has not been as widely reported, it has been postulated as an explanation for some aging effects in postural control (Slobounov et al., 1998). A larger comfort zone (or unregulated zone) would result in greater objectively observed intraindividual variability while subjectively perceived, self-reported variability would not show this increase since an individual would perceive themselves as being at equilibrium while in their comfort zone. This difference between subjective and objective variability provides a possible way of functionally modeling the dynamics of observed age-related changes in variability.

One way of interpreting voluntary selection within the framework of dynamical systems is as a choice of equilibrium as a goal state from a range of optional equilibria. For instance, an individual might decide to pursue a hobby such as to learn to play the piano. Time spent practicing the piano may contain both rewards and frustrations, providing feedback that results in a self-regulation of piano playing time investment. But this is time that cannot be spent swimming or riding a bicycle, so other goals such as maintaining a healthy body may also contribute to the optimal amount of piano-playing time — the equilibrium value. Thus, an individual may not only select an equilibrium type (piano playing), but also the level of the equilibrium value given other constraints.

An example of loss-based selection, such as gait, could be cast within the framework of attractor basins as shown in the progression from middle adulthood to old age in the hypothetical data plotted in Figure 3. Some basins of attraction may no longer be available and so choices must be made from a restricted set of options. In young and middle adulthood, one might be able to walk, skip, hop on one leg, jump, or run while traversing an obstacle course. But in late life, due to decreased muscle mass, reduced peripheral sensation, and visual limitations not all of these options may be available. One's choice of gait might be reduced to simply walking. Each of these gaits can be considered to be a periodic attractor, and thus in late life a set of bifurcations may occur that aggregate many periodic attractors into a smaller set and thus reduce the opportunities for selection. As the reduction in attractors occurs, individuals may choose to optimize their energy expenditure within the attractors remaining. But eventually the remaining choices of gait may require compensation such as a cane or walker.

Infants are at the other end of this continuum. As their muscles increase in strength, new gaits become available to them. Thelen's work on the dynamics of infant locomotion has demonstrated that, prior to development of sufficient muscle tone to support standing, an infant can walk on a treadmill in a tank of water that comes up to the infant's waist. The water provides sufficient buoyancy for the infant's legs so that the periodic attractor of walking emerges (Thelen & Smith, 1994). Thelen & Smith argue that this provides evidence of developmental bifurcation in gait attractors. It is not much of a stretch to hypothesize that a similar reduction in gait attractors occurs in late life. And just as we expect greater

variability in infants' gait as new attractors form, we would expect there to be greater variability and instability in gait as attractors are lost.

Developmental change may be observed in the value of the state variable at the centroid of the basin of attraction, i.e., the equilibrium value. Suppose the hypothetical example shown in Figure 5 depicts positive affect. Whereas in adolescence the individual regulates around lower levels of positive affect, from adulthood on the individual might regulate towards relatively higher levels. This type of intraindividual change in equilibrium values does not appear to be well-captured within the language of selection, optimization, and compensation, although one might characterize it as a gradual developmental change in goal *level* rather than a matter of selection between options.

When longitudinal data are modeled using latent growth curves (e.g., McArdle & Hamagami, 1992), the results provide information about intraindividual change in point equilibrium value. This comes about due to the fact that latent growth curve models are typically fit as intercept, slope and curvature models. A latent growth curve model thus makes an implicit assumption that the basin of attraction is a parabolic bowl for a regulatory process with a point equilibrium — At each age, for each individual, there is one best point estimate for the latent growth curve. In addition, when the values of a state variable are fluctuating around an equilibrium and are fit by this type of model, observed variability becomes part of the error variance. Thus, individuals with greater amplitude fluctuations are simply depicted as not fitting the model as well as those with lower amplitude fluctuations. On the other hand, when fluctuating data are fit using dynamical systems models for processes, hypotheses concerning developmental processes can be tested since the fluctuations inform the model for the process: larger fluctuations may fit a more shallow attractor for one individual while smaller fluctuations fit a more steep attractor for another individual.

Differentiating between intraindividual change in equilibrium and intraindividual change in dynamics within a model allows separate estimation of an individual's equilibrium level relative to the population, i.e., an objective assessment of a variable and an individual's level relative to their own regulatory equilibrium, i.e., subjective assessment of a variable. By separating these two components, a model can be constructed to estimate functional relationships between an individual's change in equilibrium level and their regulatory dynamics.

Individual Differences and Intraindividual Change in Regulation

The third and fourth categories of dynamics we will consider are individual differences in regulation and intraindividual change in regulation. This is where the dynamical systems framework is particularly advantageous, since patterns in short term variability can be modeled in the same context as long term changes in equilibria.

Individual differences in regulation are, in dynamical systems terms, differences in the geometry of attractors. While one individual may regulate very closely around an equilibrium value, producing a very steep attractor basin, another individual may not exhibit nearly as much regulatory behavior, resulting in a relatively shallow basin. Steep basins are associated with fast fluctuations relative to shallow basins. As an example, in a study of recently bereaved widows, family members reported perceived control in the widows that was associated with the steepness of the basin of attraction: steeper basins (i.e., faster

fluctuations) were associated with lower levels of perceived control (Bisconti et al., 2006).

The degree of individual differences in attractor geometry may also be related to age. For instance, one possible reason that greater variance and variability is observed in many variables in older populations may be due to greater individual differences in the shape of the attractors and/or more shallow attractor basins. Using dynamical systems models, these become testable hypotheses. If greater individual differences observed in older populations is due to more shallow attractor shape in older individuals than in younger individuals, then this age-related difference will show up in group differences in the parameters of the differential equations fit to each group. To the extent that age-related increase variability is due to increased individual differences in equilibrium values the age-related difference will show up in the variance of the latent equilibrium values, i.e., the intercept terms of the equilibria.

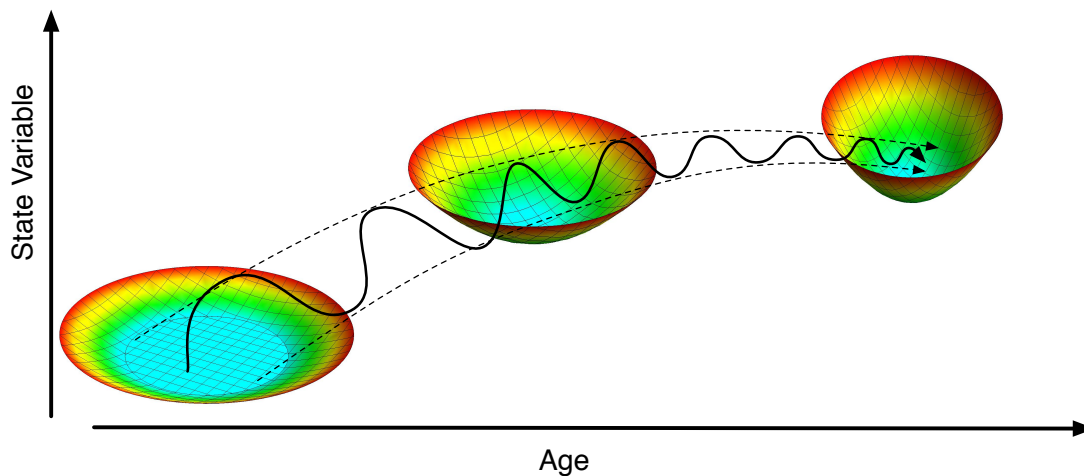


Figure 5. Age-related changes in variability in behavior or performance characterized as developmental change in basins of attraction. Note that both decreased amplitude of fluctuations as well as higher speed fluctuations are associated with the narrowing of these attractor basins.

Developmental change in equilibrium dynamics may be modeled and visualized as changes in the geometry of basins of attraction. Figure 5 shows a graphical representation of a hypothetical individual for whom a state variable (e.g., positive affect) is regulated as a flat-bottomed comfort zone in youth becomes regulated with more strength in middle age and finally exhibits a high degree of regulation in older age. In this case, we would predict decreasing range of the variable in old age as the areas in blue become larger — the older individual in this figure feel more need to exert regulation over behavior because he or she would be less comfortable with a wide range of values of behavior before regulation would be required. Please note that this example is only for didactic purposes and is not empirically estimated. The literature is, as yet, lacking good examples of longitudinal change in individual dynamics. Note that this sort of developmental change in basin of attraction bears more than passing resemblance to the notion of an epigenetic landscape first proposed by Waddington (1957) as a theory for gene-environment interactions.

Individual differences and intraindividual change in regulation can be described as optimization of methods for achieving a goal. Optimization in this regards refers to the idea that regulatory methods may be chosen and emphasized so as to optimize the geometry of an attractor basin around a selected goal, an equilibrium. An individual can have control of her or his goal, select the methods by which to regulate, and select the amount of regulation to exert. Thus the dynamical systems view adds a new dimension to the idea of optimization: it is not only important to understand which methods are being employed, but also the parametric degree to which they are emphasized and the functional form the dynamic regulation takes (e.g., the differences between Figures 1–a, b, c, and d.

Within a dynamical systems framework, selection and optimization take on additional nuanced meaning. Selection can be selection of equilibrium (i.e., goal), selection of optimization method, as well as degree to which that optimization is applied. Optimization is represented by the shape of the regulatory attractor, but the attractor shape can change over time and potentially undergo bifurcation. In turn, as described previously, bifurcation will imply both new opportunities for equilibrium value selection as well as necessary changes in attractor strength during the bifurcation process.

An example hypothesis related to bifurcation and intraindividual change in regulation is that transition from in-home living to an assisted living facility would be expected to be accompanied by temporary reduction in self-regulation of affect. The introduction of a set of new attractors (the opportunities for daily routines in the assisted living facility) would necessarily require extinguishing the previous set of attractors (daily in-home routines) and during that bifurcation process, a shallow attractor surface would form, in turn implying greater variability in regulatory processes. To the extent that affect self-regulation is coupled with daily routines, highly variable affect during the transition would be expected. This is a testable hypothesis using differential equation models for affect regulation.

Stable Regulatory Dynamics with Change in Behavioral Indicators

There are many instances where we could expect to see intraindividual change in equilibria and/or change in regulation coupled to the environment. For instance, environmental contexts tend to change over time. Environmental change might be continuous over a short or long time-frame, or change might also be discontinuous. Continuous short-term environmental changes include diurnal, weekly, and monthly cycles as well as job or social stressors and rewards. Longer term continuous environmental changes can include seasonal cycles, accrual of savings, or age-related decline in cognitive abilities. As examples of discontinuous change, a person might change marital status, move from one part of a city to another, change jobs, retire, or move to an assisted living facility. Social and family environments may change — children leave home, marry, and start families of their own. A spouse might pass away, leaving an individual as a widow or widower. Physical challenges can also be the source of environmental change: an individual might break a leg and thus need to find alternate methods of transportation, a heart attack or other disease might constrain exercise options, or onset of diabetes might require changes in diet.

In response to environmental changes, individuals may regulate their behavior using intrinsic dynamic mechanisms as described in the two preceding sections. However, when environmental change affords entirely new opportunities or removes the possibility of continuing existing habits, individuals can adapt by making fundamental changes to their habitual

behavior. Behaviors that had been used as a methods for regulation may be dropped and other behaviors selected and habitualized.

These ideographic dynamic changes can be described within the framework of selection, optimization, and compensation. However, selection, optimization, and compensation are frequently characterized in terms of discontinuous change rather than within the context of continuous adaptation between behavioral indicators. That is to say, part of a regulatory strategy relative to an equilibrium might include adapting which behaviors are applied. For instance, a goal of health through exercise might be maintained through a variety of behaviors. By increasing the frequency of one behavior while decreasing the frequency of another behavior, one could maintain a constant regulation of cardiovascular health even though the behaviors were changing over time. Such regulation could be viewed in the framework of compensation: if one form of exercise became more difficult with age, one might continue to optimize cardiovascular health by selecting a new form of exercise. Aging knee joints might begin to preclude running while low impact exercise like swimming could substituted. This type of substitution does not imply a bifurcation of the attractor since the regulatory goal (i.e., equilibrium point) and strength of regulation (i.e., attractor basin shape) might remain constant while the observed behaviors change. The language of dynamical systems helps provide nuance to the meaning of selection, optimization and compensation.

In order to test hypotheses related to this type of behavioral substitution in the presence of stable equilibria and regulation, it is necessary to be able to fit a model in which the relationships between latent derivatives (i.e., the differential equation) remains constant while indicators for the latent variables change over time. Nesselroade and colleagues (Nesselroade, Gerstorf, Hardy, & Ram, 2007) have proposed *ideographic filtering* methods for dealing with latent variables whose manifest indicators may differ between individuals or within an individual over time. The basic premise of these methods is that the meaning of a latent variable (e.g., cardiovascular health, well-being, or quality-of-life) may remain stable, as defined by covariances between several latent variables, whereas the behaviors that people select to express the latent construct may be fungible over time. Ideographic filtering applied to dynamical systems models implies that underlying regulatory mechanisms remain invariant while the way in which dynamics are manifested may differ between individuals or within an individual over time.

Coupled Dynamics

We have as yet limited ourselves to univariate dynamics — systems in which there is only a single latent construct with intrinsic dynamics. Clearly, this is a gross simplification of the complexity of human behavioral and developmental processes. Two or more processes may be involved — each process regulating itself as well as co-regulating processes with which it is coupled. This is what is known as *coupled dynamics*. The coupled processes might be within one individual or between two or more individuals.

As an example of coupled processes within an individual, consider eating behavior and ovarian hormones in young women (Klump et al., in press). Daily measures of eating behavior, estradiol, and progesterone show patterns of coupling when modeled using dynamical systems analysis (Hu, Boker, Neale, & Klump, in review). In addition, individual differences in emotional reactivity appear to be related to the strength of coupling between eating and ovarian hormones. As women age, their ovarian hormone cycles change, leading

to menopause. In coupled systems, we can test hypotheses of age-related changes in both the intrinsic dynamics of each component of the system, but also in the strength of coupling.

Coupled processes may also be used to model relationships between individuals. For instance, married couples express day-to-day variability in their need for intimacy in their relationship (Laurenceau, Rivera, Schaffer, & Pietromonaco, 2004). Each partner in the couple may have an intrinsic dynamic that regulates their need for intimacy given their previous day's need, recent events external to the marriage, and their perceptions of their spouse's needs. Each married pair can be considered to be (literally) a coupled system. The coupling strengths of husband-to-wife and of wife-to-husband are parameters in which we may observe individual differences (Boker & Laurenceau, 2005). It is also likely that developmentally related changes in these coupling strengths would be observed within couples over time. As a married couple develops into older adulthood, we are also likely to see intraindividual change in the manner in which intimacy is expressed even while the pairwise coupled dynamic stays reasonably stable.

One exciting application of coupled dynamics is to systems with multiple time-scales. For instance, short-term regulation in stress may be coupled with longer-term developmental change in health outcomes (Ong et al., 2009). Rather than looking at a simplistic model where the level of a point equilibrium in stress is predictive of a later health-related equilibrium level, a dynamical systems model uses the means and covariances of equilibrium levels, regulatory mechanisms, and coupling strengths to test hypotheses. Thus, a model can be fit in which developmental change in health has an effect on stress regulation while simultaneously stress regulation has an effect on developmental change in health. Coupled feedback hypotheses in multi-timescale relationships between variables such as health and stress are both reasonable and testable using dynamical systems analysis.

Methods and Models for Dynamical Systems Analysis

There is a rapidly growing literature describing methods for specifying and fitting dynamical systems models. While the scope of the current article precludes an in-depth introduction to these methods, a short introduction and guide to some of the relevant literature follows. In this section we also present a grid of some of the possible dynamical systems modeling methods that could be applied for testing hypotheses about selection, optimization and compensation.

The basic idea behind a dynamical systems model is that one or more equations are specified that relate the time derivatives of selected variables to one another; that is to say, one or more differential equations are specified. For instance, if the slope with respect to time of a variable is proportional to its distance from equilibrium, a first order linear differential equation results,

$$\dot{x} = \zeta x + e, \tag{1}$$

where \dot{x} is the first derivative of x with respect to time. When ζ is negative, the resulting dynamic is one of exponential decay (see Boker, 2012, for a short introduction). Second order linear differential equations,

$$\ddot{x} = \zeta \dot{x} + \eta x + e, \tag{2}$$

allow for oscillation about an equilibrium in a manner that can model resiliency with characteristics of elasticity (see Boker, Montpetit, Hunter, & Bergeman, 2010, for a discussion

of resiliency, elasticity, and differential equations). Linear coupled systems can be modeled as systems of first and second order equations (see Boker & Laurenceau, 2007; Hu et al., in review, for examples). More complex models include examples of nonlinear differential equations (Thompson & Stewart, 1986).

In order to test a dynamical systems hypothesis, repeated observations are required for each participant in a study. These repeated observations could range from time series output by physiological measurement devices to daily diary questionnaires to longitudinal observations over a series of years. There must be sufficiently intensive measurement of each individual in order to identify individual differences in dynamics. The number of observations required per individual will depend on the internal reliability of measurement device or scale, the amount of variability shown by each individual, the effect sizes of the parameters of the differential equation model, and how well the time scale of measurement matches the time scale of the process. The first of these three requirements are familiar to those who have performed power analyses. However, the last requirement, known as the Nyquist Limit (Luke, 1999), is unique to dynamical systems modeling. An incorrectly specified interval of measurement can completely miss a fluctuating process (Nesselroade & Boker, 1994). In practice, differential equation models of psychological processes have identified reliable individual differences in dynamics with as few as 25 observations per individual (Bisconti et al., 2006). But note that when estimating differential equations, the power obtained by adding an additional repeated observation for each person can greatly outweigh the power obtained by adding the same number of observations spread across more participants (Oertzen & Boker, 2010).

There are a wide variety of estimation methods for obtaining parameter estimates for differential equation models. One class of methods involves difference scores (e.g., McArdle, 2001). A second class of models use continuous time specifications of the integral form of the differential equation with nonlinear constraints on parameters (e.g., Harvey, 1989; Oud & Jansen, 2000; Singer, 1993). Another class of models estimate a latent variable version of the differential equation (e.g., Boker, Neale, & Rausch, 2004; Boker, 2007) using a convolution filter (Savitzky & Golay, 1964) and structural equation modeling software. Latent differential equations have the advantage of being relatively easy to specify and can be extended to models including moderation, coupling, categorical variables, or mixture distributions.

Summary

Successful developmental processes have been described in terms of maximization of gains, minimizations of losses, and resilient maintenance in a dynamic conditioned by age, cultural, and personal factors (P. B. Baltes et al., 1999). To put this in terms of continuous time dynamical systems, *gains* are positive first derivatives with respect to time, *losses* are negative first derivatives with respect to time, *maintenance* is an attractor basin around an equilibrium, and *conditioning* by age, cultural, and personal factors are moderators of the covariance relations between the derivatives that shape the attractor basin at any one moment in time.

Developmental and regulatory processes modeled using dynamical systems analysis can also be considered within the framework of selection, optimization, and compensation. Figure 4 summarizes this relationship in a grid in which individual differences and intrain-

dividual change from the point of view of dynamics are crossed with selection, optimization, and compensation. This grid differentiates between sources of process-oriented variance as a step towards implementing theories of development and regulation as testable models.

By focusing on the ways in which individuals may differ or change in their regulatory dynamics, the grid in Figure 4 emphasizes important categories in the way that selection, optimization, and compensation occur. For instance, selection may occur as it is commonly phrased: as a selection of behaviors in which an individual engages. But individuals can also select which regulatory strategy they wish to use or which of several equilibria is preferred. One obvious application of selection to coupled systems is assortative mating: a choice with long term developmental consequences.

Similarly, optimization is commonly thought of in terms that correspond to a dynamical systems view of intrinsic regulation with respect to an equilibrium. However, optimization may also be evident in long-term change in an equilibrium set as previously illustrated in Figures 3 and 5. Optimization can also occur as incremental adjustments of importance (i.e., weights) associated with particular behaviors, resulting in a time-varying measurement model for latent constructs. For instance, as an individual ages, he or she might place less emphasis on work and more on family (or vice versa) in order to optimize his or her definition of quality of life. Resilience could be framed as optimization of coupling between short-term stress regulation processes and long-term developmental changes associated with aging — an example of optimization of coupled systems.

Compensation requires some change external to the regulating system, (formally termed a *perturbation*), to which the regulatory system responds in a compensatory manner. Compensation as commonly considered is, in dynamical systems terms, a regulation response to an external perturbation. However, an external perturbation may also influence the system by effecting change in the shape and/or location of a basin of attraction about an equilibrium. In this case, the perturbation could be modeled as having produced a moderating effect on either the parameters of the dynamical system or on the location of the equilibrium. The perturbation could also produce a moderating effect on loadings of a measurement model or on the coupling strength between two or more coupled systems.

Selection, optimization, and compensation are concepts that have gained considerable traction in the adult development and aging literature. One reason that these ideas hold appeal is that they describe processes that happen to individuals: and thus have the potential to help understand the etiology of developmental changes. Selection, optimization, and compensation are fundamentally dynamic concepts. We have a brief introduction to the ideas inherent in dynamical systems analysis: specifically equilibria, basins of attraction, and differential equation models. These dynamical systems concepts fit well with a process-oriented theory of selection, optimization and compensation. By focusing on how individuals' dynamics may differ from each other and change over time, we have proposed a way to improve the specificity with which individual differences in selection, optimization, and compensation can be categorized. Dynamical systems modeling of individual differences and intraindividual changes in equilibria and in attractor dynamics provides a way to integrate subjective experience of regulation relative to a set of goals and the functional resources and activities required to maintain stability.

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