Growth and Welfare Gains from Financial Integration Under Model Uncertainty[*]

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February 14, 2019

Abstract

We build a robustness (RB) version of the Obstfeld (1994) model to study the effects of financial integration on growth and welfare. Our model can account for the empirically observed heterogeneity in the relationship between growth and volatility for different countries. The calibrated model shows that financial integration leads to significantly larger gains in growth and welfare for advanced countries than developing countries, with some developing countries experiencing growth and welfare loss in financial integration. Our analytical solutions help uncover the key mechanisms by which this happens.

JEL Classification Numbers: *C61, D81, E21.*

Keywords: Robustness, Model Uncertainty, Financial Integration, Risk Sharing, Economic Growth, Welfare.

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1 Introduction

This paper builds an endogenous growth model with portfolio choices to study effects of financial integration on growth and welfare for different countries. The main goal is to reconcile two contradicting research lines in the literature on growth and volatility by introducing model uncertainty and re-evaluate the growth and welfare implications of financial integration under model uncertainty. The first research line is theoretical and led by Obstfeld (1994) which proposes a general equilibrium version of the Merton-type model to link the financial diversification of an economy (openness) to long-run growth. In his model, an economy with an interior portfolio of risky and riskless capital (a diversified equilibrium) will unambiguously generate a positive relationship between the average growth rate and the volatility of real GDP per capita while an undiversified equilibrium (with only risky capital) can generate a negative relationship provided the intertemporal elasticity of substitution exceeds one.

The second research line, led by recent empirical studies on growth and volatility, challenges the theoretical predictions above. Ramey and Ramey (1995) document a robust negative relationship between the average growth rate of an economy and the volatility of output; this relationship holds after controlling for a number of country-specific factors.[1] More recently, Kose, Prasad, and Terrones (2005, henceforth KPT) further show that the growth-volatility relationship differs in different groups of countries – it is negative in some developing countries but positive in industrial countries. Using more recent data over the period $1962-2011$ from the Penn World Tables 8.0, we find these relationships continue to hold (see Figures 3 - 5).[2] Furthermore, cross-country holdings of US government debt (essentially a risk-free asset) are large and widespread, indicating that an undiversified equilibrium discussed in Obstfeld (1994) does not look empirically plausible.[3]

The idea of reconciling the above two contradictive findings by introducing model uncertainty is motivated by recent literature that highlights the importance of model uncertainty in explaining household decisions and aggregate dynamics. First, the recent literature suggests that

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[1]See Aghion and Banerjee (2005) for a recent survey on volatility and growth.

[2]KPT (2005) divide developing countries into two groups - less financially integrated (LFI) countries and more financially integrated (MFI) countries. Their regressions show that the positive relationship between growth and volatility for MFI countries is not statistically significant. As shown in the empirical section, our data with more years make it statistically significant. In this paper, we use "advanced countries" and "industrial countries" interchangeably, in contrast with developing countries.

[3]As we will discuss below, the theoretical prediction on the negative volatility-growth relationship depends on the magnitude of the elasticity of intertemporal substitution (EIS). Specifically, to generate a negative relationship, the value of the EIS in our model needs to be greater than one in the undiversified equilibrium. It is worth noting that the evidence on the IES is mixed. (The estimates range so widely that almost any value between 0 and 2 looks empirically reasonable.)

model uncertainty (or Knightian uncertainty) could be a crucial factor in the recent economic and financial crises. For example, Caballero and Krishnamurthy (2008) argue that the common aspects of investor behavior during the financial crisis - reevaluation of models and disengagement from risky investment - indicate agents' decisions for worst-case scenarios, meaning that these activities involved model uncertainty and not merely an increase in risk exposure. Second, the recent literature shows that introducing model uncertainty to standard models on household consumption-saving problems helps solve several classic macroeconomic puzzles such as the excess volatility puzzle (Djeutem and Kasa 2013), the current account volatility puzzle (Luo, Nie, and Young 2012), and the skewed wealth inequality in the U.S. (Kasa and Lei 2017). Third, the literature also finds that welfare costs due to model uncertainty can be significant. (See, for example, Barillas, Hansen, and Sargent (2009), Ellison and Sargent (2012), and Luo, Nie, and Young (2019)). These studies suggest model uncertainty is an important factor influencing household decisions (and therefore economic growth in the current context) and has large welfare implications. Thus, our goal is to investigate whether model uncertainty can help explain the different growth-volatility relationships in developing and advanced countries and evaluate the growth and welfare effects of financial integration under model uncertainty.

Following Hansen and Sargent (2007), we introduce model uncertainty by assuming agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, agents make their decisions as though the subjective distribution over shocks was chosen by an evil agent aiming to minimize their utility. That is, agents have a preference for robust decisions (RB).[4] Under RB, agents have a reference model (called the approximating model) in their mind and consider an unstructured range of models (called the alternative models) surrounding that model. The evil agent selects the worst of these alternative models (called the distorted model). Many recent papers have shown the usefulness of viewing agents as having (potentially) misspecified models of the economy and being aware of this fact; Hansen and Sargent (2007) provide a book-length introduction and discussion of the literature. The key assumption of the RB models proposed by Hansen, Sargent, and their co-authors is that the decision-maker has created the approximation model by a specification search that they do not model and believes that data may come from an unknown member of a set of unspecified models near the approximating model. In other words, the constraint imposed on the evil agent's actions is set so that the decison-maker only hedges against models that could have plausibly

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[4]See Hansen and Sargent (2007) for a textbook treatment on robustness. It is worth noting that we can use either robust decision-making or recursive multiple-prior utility (Chen and Epstein 2002) due to ambiguity aversion to capture the same idea that the decision maker is concerned that their model is misspecified. In this paper, for tractability, we follow Hansen and Sargent (2007) and use the robust control to model concerns about model misspecification.

generated the observed data.

A desire for robust decision rules complicates the link between average growth and volatility. If we interpret the model misspecification fears as "entirely in the head" of the agents, then robustness only affects the magnitude of the correlation between volatility and growth; this model therefore cannot replicate the negative relationship between mean growth and volatility as it becomes observationally equivalent to a standard rational expectations (RE) model with a higher coefficient of risk aversion. If instead we interpret the model-misspecification fears as *justified*, which means the worst-case scenario happens and the data are generated by the distorted model, then the cutoff value for the IES to generate the negative relationship between volatility and growth is smaller than one and is decreasing with the degree of concern about model uncertainty. Under this interpretation, we can calibrate the model to capture the observed negative correlation in developing countries.

We make several comments on the two models here. First, there is no definitive justification for either perspective-Hansen and Sargent (2007) usually adopt the "entirely in the head" perspective, but estimation via an indirect reference approach, which takes into account key features in the data, suggests that the distorted model is an empirically-plausible model of the data. Put another way, once we entertain the idea that agents do not trust their models, there is no "true" model anymore. Our robustness model fits the data on volatility and growth only if the distorted model generates the data. Second, we do not consider the possibility that the agents learn the outcome of their decisions and update the approximating model. The main justification for this assumption is that the impatient agent cannot avoid facing up to her model misspecification doubts simply by waiting for enough data.[5] In addition, the distorted model is constructed using the approximating model, and these two models are very close and are statistically indistinguishable. As explained below, the probability that the two models (the approximating model and the distorted model) cannot be statistically distinguished with 50 years of data lies in the range $0.17-0.38$. Finally, in the distorted model the risk premium is low, which explains why it is a "worst case"; the agents are not being compensated adequately for the risk that they bear.

This paper provides three main findings. First, we derive analytical solutions and show conditions under which the growth rate and volatility of real GDP are negatively correlated in a diversified equilibrium (the economy holds a mix of risky and riskless capital) if

$$\vartheta > \frac{1+\psi}{1-\psi}\gamma,$$

where ϑ, γ, and ψ are the parameters governing the degree of robustness, the coefficient of relative

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[5]In other words, relative to her discount factor, it would take far too long for her to learn which model generated the data.

risk aversion, and the elasticity of intertemporal substitution, respectively.

Second, to discipline the values of these key parameters, we jointly calibrate these three parameters to match three key moments in the data – average output growth, the volatility of growth, and the growth-volatility correlation-for industrial, less financially integrated (LFI), and more financially integrated (MFI) countries separately.[6] Our analytical solutions greatly facilitate this calibration process. Our calibration results imply developing countries (including both LFI and MFI) face significantly greater model uncertainty and the degree of model uncertainty decreases with the degree of financial integration. Using the detection error probability (DEP) proposed by Hansen and Sargent (2007), the corresponding DEPs for LFI countries, MFI countries, and industrial countries are 0.17, 0.18, and 0.38, respectively.[7] In other words, the agents in LFI and MFI countries face greater model uncertainty and thus consider a *larger* set of models than the agents of industrial countries when making consumption and investment decisions.

Third, we quantitatively evaluate the growth and welfare gains associated with financial integration. Using our preferred specification–the calibrated distorted model–we find that growth and welfare gains differ for different groups of countries with LFI countries experiencing growth and welfare loss and the other two groups of countries (MFI and industrial) experiencing growth and welfare gains. Our closed-form solution for equilibrium growth and welfare makes it easy to explain these results. In general, our formula shows that financial integration influences growth through two channels. The first channel is to influence the equilibrium interest rate. The second channel that financial integration influences growth is through volatility. Total growth volatility is influenced by the share and volatility of the risky assets. As financial integration reduces total volatility for the international risk-asset combination (i.e., the world-wide mutual fund), it also leads to an increase in the share of the risky assets. On balance, the higher risky share and lower volatility cause the volatility of growth to increase in all three groups of countries. Finally, the difference in the growth-volatility relationship plays a key role in explaining the effect of the second channel on growth-the volatility of GDP growth is positively correlated with growth in industrial countries and MFI countries but negatively correlated with growth in LFI countries. This is why financial integration improves growth in the first two groups of countries but reduces growth in LFI countries.

Regarding the welfare implications, our formula shows that the welfare improvement due to financial integration is an increasing function of growth and the equilibrium risk-free rate. For this reason, in our model economy in which the equilibrium interest rate stays unchanged after the

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[6]To be consistent with the empirical analysis, we divide developing countries into LFI and MFI groups in the calibration.

[7]Intuitively speaking, this result means the probability that a likelihood-ratio test cannot distinguish competing models in LFI countries is 17 percent, much lower than the probability in industrial countries.

financial integration, the welfare implications are similar to the growth implications.[8] However, in an alternative calibration, we consider a scenario in which LFI countries lack safe assets by assuming the return to their (true) risk-free capital is lower than in MFI and industrial countries. Under this calibration, LFI countries experience an increase in the risk-free rate after financial integration, which improves both growth and welfare. In one quantitative example (Alternative 2 in Table 6), the positive effect of an increase in the risk-free rate on welfare in financial integration more than offsets the negative effect of a decrease in growth, which leads the welfare of LFI countries to improve as well.

Literature review. This paper is related to three branches of literature. First, as described above, it is related to the literature on robustness and ambiguity. Second, it is related to the literature on growth and volatility following Ramey and Ramey (1995). Third, it is related to the literature on growth and welfare implications of financial integration following Obstfeld (1994). Schularick and Steger (2010) show that integration can boost growth through higher investment and Eozenou (2008) presents evidence for a positive connection between integration and macroeconomic volatility. As the standard Obstfeld model predicts a positive correlation between volatility and growth, it may overstate the welfare gains associated with financial integration (see also Epaulard and Pommeret 2005). The evidence on the welfare effects of financial integration is mixed. For example, Athanasoulis and van Wincoop (2000) find that the welfare gain from risk sharing related to growth volatility is about 6.5 percent for 49 developed and developing countries. Hoxha, Kalemli-Ozcan, and Vollrath (2013) also find large gains from international financial integration even when growth rates are not affected in the long run. By contrast, Gourinchas and Jeanne (2006) show that the gains are generally smaller in models with exogenous growth, which is a manifestation of the old Lucas observation that the welfare costs of aggregate consumption risk are magnitudes smaller than the welfare costs of low growth.

The rest part of the paper is organized as follows. Section 2 presents a robustness version of the Obstfeld-type model with recursive utility in a closed economy and discusses how the presence of robustness can have the potential to generate the observed negative relationship between growth and volatility of the macroeconomy. Section 3 presents the theoretical results on growth and welfare allowing financial integration. Section 4 shows our quantitative analysis. Section 5 concludes.

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[8]In our benchmark calibrated model, the equilibrium interest rate stays unchanged after the financial integration because we assume risk-free capital has the same return in all three groups of countries.

2 A Risk-Sharing Model with Recursive Utility and Model Uncertainty

2.1 The Model Setting

Following Obstfeld (1994), in this paper, we consider a continuous-time risk-sharing model with multiple assets. Specifically, we assume that individuals save by accumulating capital and by making risk-free loans that pay a real return i_t. There are two types of capital: one is riskless capital offering a constant return (r) and one is risky capital used in the following stochastic production processing:[9]

$$dy_t = \mu k_{e,t} dt + \sigma k_{e,t} dB_t, \tag{1}$$

where dy_t is the instantaneous net output flow, $k_{e,t}$ denotes the stock of capital, $\mu > r$ is the expected rate of return to the risky capital, σ is the standard deviation of the production technology, and B_t is a standard Brownian motion defined over the complete probability space.

If $i_t < r$, there is no equilibrium because this condition implies an arbitrage profit from issuing loans and investing the proceeds in the risk-free asset. If $i_t > r$, there exists an equilibrium with no risk-free assets if and only if there exists a short sale constraint on capital, which we implicitly impose. Finally, when $i_t = r$, the division between the risk-free asset and the loan is indeterminate. Consequently, the individuals only need to choose from two assets: the risky capital and a composite safe asset offering a return i_t. Later we will show that the real interest rate (i_t) is constant in equilibrium.[10] Let $k_t = k_{e,t} + b_{f,t}$ denote total wealth, where $b_{f,t}$ is holdings of the composite safe asset. It is straightforward to show that the budget constraint for the representative consumer can be written as:

$$dk_t = [(i + \alpha_t (\mu - i)) k_t - c_t] dt + \alpha_t k_t \sigma dB_t, \tag{2}$$

where and α_t is the fraction of wealth invested in risky capital.

Following Duffie and Epstein (1992), we consider the following continuous-time recursive utility:

$$J_t = E_t \left[\int_t^\infty f(c_s, J_s) \, ds \right], \tag{3}$$

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[9]It is worth noting that the AK specification, (1), can be regarded as a reduced form of a stochastic Cobb-Douglas production function specification when labor is supplied inelastically. The proof is available from the corresponding author by request.

[10]In many ways, our model is similar to Cagetti and DeNardi (2008) in which households can choose to save in the form of riskless corporate capital, riskless bonds, or risky entrepreneurial capital. Our model is also related to Mendoza, Quadrini, and Ríos-Rull (2009) who study the connection between financial openness and gross capital positions in a model with risky assets in multiple countries. Both papers have models that are not amenable to analytical solutions.

where J is continuation utility and $f(c, J)$ is a normalized aggregator of current consumption and continuation utility given by:

$$f(c, J) = \frac{\delta(1-\gamma)J}{1-1/\psi}\left[\left(\frac{c}{[(1-\gamma)J]^{1/(1-\gamma)}}\right)^{1-1/\psi} - 1\right],\tag{4}$$

where $\delta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution. When $\psi = 1/\gamma$, the above recursive utility reduces to the standard time-separable power utility. As $\psi \to 1$, J converges to:

$$f(c, J) = \delta(1-\gamma)J\left[\ln(c) - \frac{1}{1-\gamma}\ln((1-\gamma)J)\right].\tag{5}$$

The only difference from the standard additive power utility case is that here we use the aggregator $f(c, J)$ to replace the instantaneous utility function $u(c)$ from the standard expected utility case.

2.2 Introducing Model Uncertainty due to RB

To introduce robustness into the above model, we follow Anderson, Hansen, and Sargent (2003) (henceforth, AHS) and Maenhout (2004) and introduce a preference for robustness (RB) by adding an endogenous distortion $v(k_t)$ to the law of motion of the state variable k_t:

$$dk_t = [(i + \alpha_t(\mu - i))k_t - c_t]dt + \alpha_t\sigma k_t(\alpha_t\sigma k_t v(k_t)dt + dB_t).\tag{6}$$

As shown in AHS (2003), the objective $\mathcal{D}J$ defined in (8) plays a crucial role in introducing robustness. $\mathcal{D}J$ can be thought of as $E[dJ]/dt$ and is easily obtained using Itô's lemma. A key insight of AHS (2003) is that this differential expectations operator reflects a particular underlying model for the state variable. The consumer accepts the approximating model, (2), as the best approximating model but is still concerned that it is misspecified. She therefore wants to consider a range of models (i.e., the distorted model, (6)) surrounding the approximating model when computing the continuation payoff. A preference for robustness is then achieved by having the agent guard against the distorting model that is reasonably close to the approximating model. Under RB, the Hamilton-Jacobi-Bellman (HJB) equation can be written as:

$$\sup_{c_t, \alpha_t}\inf_{v}\left[f(c_t, J_t) + \mathcal{D}J(k_t) + v(k_t)(\alpha_t k_t\sigma)^2 J_k + \frac{1}{2\vartheta(k_t)}(\alpha_t k_t\sigma)^2 v(k_t)^2\right].\tag{7}$$

where

$$\mathcal{D}J(k_t) = J_k[(i + \alpha_t(\mu - i))k_t - c_t] + \frac{1}{2}J_{kk}\sigma^2\alpha_t^2 k_t^2.\tag{8}$$

Note that here following Hansen and Sargent (2011) and Kasa and Lei (2017), we scale the robustness parameter $(\vartheta(k_t))$ by the sampling interval (Δt), effectively giving the agent a stronger preference for robustness as the sampling interval shrinks.[11]

In the robust control problem, the drift adjustment $\upsilon(k_t)$ is chosen to minimize the sum of the expected continuation payoff but adjusted to reflect the additional drift component in (6), and of an entropy penalty, i.e., the last three terms in (7). Note that the second and third terms in (7) are the expected continuation payoff when the state variable follows (2), i.e., the alternative model based on drift distortion $\upsilon(k_t)$.[12] $\vartheta(k_t)$ is fixed and state independent in AHS (2003), whereas it is state-dependent in Maenhout (2004). The key reason to replace fixed ϑ with a state-dependent counterpart $\vartheta(k_t)$ in Maenhout (2004) is to assure the homotheticity (scale invariance) of the decision problem, a property that is required for the model to display balanced growth. As emphasized in AHS (2003) and Maenhout (2004), the last term in the HJB above is due to the agent's preference for robustness and reflects a concern about the quadratic variation in the partial derivative of the value function weighted by the robustness parameter, $\vartheta(k_t)$. The following proposition summarizes the solution.

Proposition 1 *Under RB, the portfolio rule is:*

$$\alpha^* = \frac{\mu - i}{\widetilde{\gamma}\sigma^2}, \tag{9}$$

the consumption function is:

$$c_t^* = \text{mpc} \cdot k_t, \tag{10}$$

the evolution of risky capital for the approximating model and the distorted model are:

$$\left(\frac{dk_t}{k_t}\right)^a = \left[\psi(i - \delta) + (1 + \psi)\frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2}\right]dt + \alpha^*\sigma dB_t, \tag{11}$$

and

$$\left(\frac{dk_t}{k_t}\right)^d = \left[\psi(i - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right)\frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2}\right]dt + \alpha^*\sigma dB_t, \tag{12}$$

respectively, where

$$\text{mpc} = \psi\left\{\delta - \left(1 - \frac{1}{\psi}\right)\left[i + \frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2}\right]\right\} \tag{13}$$

is the marginal propensity of consumption to wealth and the effective coefficient of absolute risk aversion $\widetilde{\gamma}$ is defined as: $\widetilde{\gamma} \equiv \gamma + \vartheta$.

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[11]See also Hansen and Miao (2018) for a discussion on distinguishing ambiguity from risk in the continuous time limit.

[12]Note that $\vartheta(k_t) = 0$ here corresponds to the expected utility case.

Proof. See Online Appendix A. ∎

Following Obstfeld (1994), we first consider a closed-economy equilibrium in which the two capital goods can be interchanged in one-to-one ratio and the amount of asset supply can always be adjusted to accommodate the equilibrium asset demand, (9). There are two types of equilibrium: (i) one in which both types of capital are held (*diversified*) and (ii) one in which only risky capital is held (*undiversified*). In the diversified equilibrium, the interest rate i is equal to r and $\alpha^* = \frac{\mu - r}{\widetilde{\gamma}\sigma^2} \leq 1$. In the undiversified equilibrium, $\frac{\mu - r}{\widetilde{\gamma}\sigma^2} > 1$, so the interest rate is given by:

$$i = \mu - \widetilde{\gamma}\sigma^2 > r. \tag{14}$$

Since $\widetilde{\gamma} = \gamma + \vartheta > \gamma$, the equilibrium interest rate under RB is lower than that under RE. The intuition is simple: the additional amount of precautionary savings due to robustness drives down the equilibrium interest rate. The proposition summarizes the results about the expected growth rate under RB in the two equilibria.

2.3 Implications for the Relationship between Volatility and Growth

The following proposition summarizes the results about the expected growth rate in the two types of equilibria.[13]

Proposition 2 *Under RB, in the diversified equilibrium, the expected growth rate is:*

$$g^a = \psi(r - \delta) + \frac{(1 + \psi)\widetilde{\gamma}}{2}\Sigma^2 \tag{15}$$

under the approximating model and:

$$g^d = \psi(r - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right)\frac{\widetilde{\gamma}}{2}\Sigma^2 \tag{16}$$

under the distorted model. The value function is:

$$J(k) = \Omega\frac{k^{1-\gamma}}{1 - \gamma}, \tag{17}$$

where $\Sigma = \alpha^*\sigma$, $\Omega = \left(\frac{2\psi\delta + (1-\psi)(g+r)}{1+\psi}\right)^{\frac{1-\gamma}{1-\psi}}$ and $g = g^a$ (g^d) under the approximating model (the distorted model).

In the undiversified equilibrium, the expected growth rate is:

$$g^a = \psi(\mu - \delta) + \frac{1}{2}(1 - \psi)\widetilde{\gamma}\Sigma^2 \tag{18}$$

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[13]Note that the two types of equilibria do not co-exist; the economy only has one equilibrium for a given set of parameters.

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under the approximating model and:

$$g^d = \psi (\mu - \delta) + \frac{1}{2} \left(1 - \psi - \frac{2\vartheta}{\gamma + \vartheta} \right) \widetilde{\gamma} \Sigma^2 \tag{19}$$

under the distorted model. The corresponding value function is:

$$J(k) = \Omega \frac{k^{1-\gamma}}{1-\gamma}, \tag{20}$$

where $\Omega = (\mu - g)^{\frac{1-\gamma}{1-\psi}}$ *and* $g = g^a$ (g^d) *under the approximating model (the distorted model).*

Proof. See Online Appendix A. ∎

Before we examine the implications of RB for the relationship between volatility and growth of real GDP, we first discuss the model's predictions on this relationship under RE. When $\vartheta = 0$, the RB solution reduces to the standard RE solution, and the approximating and distorted models are coincident. Specifically, under RE, the expected growth rate in the diversified equilibrium is:

$$g = \psi (r - \delta) + (1 + \psi) \frac{(\mu - r)^2}{2\gamma\sigma^2}, \tag{21}$$

while it is:

$$g = \psi (\mu - \delta) + \frac{1}{2} (1 - \psi) \gamma\sigma^2 \tag{22}$$

in the undiversified equilibrium. Comparing (21) with (22), it is clear that the effects of the volatility of the fundamental shocks (σ^2) on the growth rate (g) are different in the two equilibria. In the diversified equilibrium, it is immediately apparent that:

$$\frac{\partial(g)}{\partial(\gamma\sigma^2)} < 0,$$

where $\gamma\sigma^2$ measures the market value of uncertainty about the return on the risky capital facing the agent. In contrast, in the undiversified equilibrium, the effect of volatility on the growth rate depends on the value of the elasticity of intertemporal substitution. Specifically, a fall in σ^2 increases growth if $\psi > 1$ but lowers growth when $\psi < 1$. The magnitude of the EIS (ψ) is a key issue in macroeconomics and finance. We can find examples in the literature that find values for ψ that range well below 1 (Campbell 1999) to well above 1 (Gourinchas and Parker 2002); Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). More recently, Havránek (2015) surveys the vast literature and suggests that a range around 0.3-0.4 is appropriate after correcting for selective reporting bias, while Crump *et al.* (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations (SCE). There does not seem to be

much consensus here, despite the clear importance of this parameter in growth models (Lucas 1990).

Now we can connect GDP volatility to growth. The standard deviation of the growth rate of real GDP, Σ, can be written as $\Sigma = \alpha^* \sigma$. Using these relationships, in the diversified equilibrium, the expected growth rate can be written as:

$$g = \psi (r - \delta) + \frac{(1 + \psi) \gamma}{2} \Sigma^2. \tag{23}$$

Note that the relationship between average growth and GDP volatility is unambiguously positive, in direct contrast to the data we examine below. However, in the undiversified equilibrium, we have $g = \psi (\mu - \delta) + (1 - \psi) \gamma \Sigma^2 / 2$, which means that there is a negative growth-volatility relationship only if $\psi > 1$ (since $\alpha^* = 1$ GDP volatility equals shock volatility). If we look at the data, it seems to us that an undiversified equilibrium is not reasonable. For example, in the US, the stock of government debt has historically hovered around 50 percent of GDP and is currently at close to 100 percent; US debt is generally considered as close to risk free as any asset and is widely held across the world (see Figure 6, which is taken from US Treasury data on foreign holdings of US government debt; we eliminate countries that lack data for June 2016). Once we rule out undiversified equilibria, the basic Obstfeld model does not replicate the negative correlation between growth and volatility.

Under RB, in the diversified equilibrium, as shown in (15), the expected growth rate under the approximating model is decreasing with Σ^2 and γ for any value of ψ. Furthermore, we also have:

$$\frac{\partial g^a}{\partial \vartheta} < 0.$$

We can see from these results that the stronger the degree of model uncertainty, the more negative the correlation between the volatility of the fundamental shock and economic growth. It is clear from (16) that, under the distorted model, the growth rate is decreasing with Σ^2 when $\psi < \frac{\vartheta - \gamma}{\gamma + \vartheta}$ and is increasing with Σ^2 when $\psi > \frac{\vartheta - \gamma}{\gamma + \vartheta}$. Furthermore, under the distorted model, we can also conclude that:

$$\frac{\partial g^d}{\partial \vartheta} < 0$$

because $\frac{\partial (-\vartheta / (\gamma + \vartheta))}{\partial \vartheta} < 0$ and $\frac{\partial \tilde{\gamma}}{\partial \vartheta} > 0$. From (17), we can see that the volatility of GDP growth, Σ^2, influences the lifetime utility only through its effect on the growth rate, g. Specifically, in the diversified equilibrium, it is straightforward to show that given the initial level of k:

$$\text{sign} \left(\frac{\partial J}{\partial \Sigma^2} \right) = \text{sign} \left(\frac{\partial g}{\partial \Sigma^2} \right),$$

where $g = g^a$ or g^d, for any value of γ, ψ, and ϑ because $\frac{\partial J}{\partial g} > 0$. The upper panels of Figure 7 show that both the growth rate (g) and lifetime utility measured by Ω are decreasing with the

degree of RB, ϑ under both the approximating and distorted models for given γ and k_0. It is clear from these two panels that the economy would experience much lower economic growth and lower lifetime welfare if it is governed by the distorted model rather than the approximating model. For example, under the diversified equilibrium, when $\gamma = 3$, $\psi = 0.63$, and $\vartheta = 3$, the growth rate is 4.75 percent in the approximating model while it is 3.39 percent in the distorted model.[14]

In the undiversified equilibrium, under the approximating model, a fall in Σ^2 increases growth when $\psi > 1$ but lowers it when $\psi < 1$. In contrast, under the distorting model, a fall in Σ^2 increases growth when $\psi > 1 - \frac{2\vartheta}{\gamma+\vartheta}$ but lowers it when $\psi < 1 - \frac{2\vartheta}{\gamma+\vartheta}$. In other words, the presence of RB weakens the condition on ψ such that economic growth is inversely related to fundamental uncertainty. When the preference for RB is strong enough, a small value of ψ can still guarantee the inverse relationship between growth and volatility. Figure 8 illustrates the inverse relationship between EIS (ψ) and RB (ϑ) for different values of risk aversion when $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta} = 0$. It clearly shows that the critical value of ψ for generating the negative relationship between volatility and growth decreases with the value of ϑ.

In the undiversified equilibrium, we can see from (20) that the volatility of GDP growth, Σ^2, also influences the lifetime utility only through its effect on the growth rate, g. It is straightforward to show that given the initial level of k, when $\gamma > 1$:

$$\text{sign}\left(\frac{\partial J}{\partial \Sigma^2}\right) = -\text{sign}\left(\frac{\partial g}{\partial \Sigma^2}\right) \text{ if } \psi < 1;$$
$$\text{sign}\left(\frac{\partial J}{\partial \Sigma^2}\right) = \text{sign}\left(\frac{\partial g}{\partial \Sigma^2}\right) \text{ if } \psi > 1.$$

where $g = g^a$ or g^d, because $\frac{\partial J}{\partial g} > 0$ for any value of ϑ. The lower panels of Figure 7 plot how the growth rate and lifetime utility vary with the degree of RB in the undiversified equilibrium when $\psi = 0.63$. They clearly show that the growth rate is increasing with ϑ and lifetime utility is decreasing with ϑ under the approximating model whereas the growth rate is decreasing and lifetime utility is increasing under the distorted model, given γ and k_0. This result is not surprising because RB affects the growth rate via two channels: (i) increasing the effective coefficient of relative risk aversion $\widetilde{\gamma}$ and (ii) reducing the $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta}$ term in the distorted model.

It is helpful at this point to illuminate exactly how the approximating and distorted models differ. To this end, we can substitute the worst-case shock solution υ into the law of motion for capital to obtain

$$dk_t = \left[\left(i + \alpha_t \frac{\gamma}{\gamma+\vartheta}(\mu - i)\right)k_t - c_t\right]dt + \alpha_t \sigma k_t dB_t.$$

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[14]We also set $r = 0.04$, $\mu = 0.082$, and $\sigma = 0.02$ for this exercise. See Section 4.2 for a detailed discussion on the calibration of the model.

Compared to the approximating model, we can therefore interpret the distorted model by absorbing the distortion term $\frac{\gamma}{\gamma+\vartheta} < 1$ into the risk premium; that is, the agents fear they live in a world with low compensation for risk.

The differing effects of ψ under the approximating and distorted models lead us naturally to consider how to view the fears expressed by agents in the model. One possible interpretation is that these fears are unjustified (they are "entirely in the head" of the agents); in this case, which is the usual one applied by Hansen and Sargent (2007), the connection between volatility and growth of real GDP is unambiguous for diversified economies and is unambiguous for undiversified economies once we know the value of ψ. However, the distorted model cannot be easily dismissed as a description of the world; calibration of ϑ via detection error probabilities (as advocated by Hansen and Sargent 2007) implies that agents would not reject the hypothesis that the distorted model describes the data at a given level of confidence. Thus, the fears expressed by the distorted model need not be dismissed as imaginary.

The following proposition summarizes the relationship between γ and ϑ in the RB model:

Proposition 3 *In the RB version of the Obstfeld model, the parameters governing risk aversion and uncertainty aversion, γ and ϑ, are observationally equivalent in the sense that they lead to the same growth rate and lifetime utility if the true economy is governed by the approximating model. In contrast, the observational equivalence does not hold if the true economy is governed by the distorted model.*

Proof. The proof is straightforward by inspecting Equations (15), (16), (18), and (19). ∎

From (9) and (10), it is clear that the RB model with the coefficient of relative risk aversion γ and the degree of RB ϑ and the FI model with $\widetilde{\gamma} = \gamma + \vartheta > \gamma$ are observationally equivalent in the sense that they lead to the same consumption and portfolio rules.[15] However, the two model economies lead to different state transition dynamics when the true economy is governed by the distorted model.

3   International Integration Under RB

In this section, we extend the above benchmark closed-economy model to a multi-country economy. Following Obstfeld (1994), we now assume that there are N countries, indexed by $j = 1, 2, \cdots, N$, and the representative agent in country j has a coefficient of relative risk aversion γ_j, an elasticity of intertemporal substitution ψ_j, a discount rate δ_j, and a parameter governing the implied preference for robustness ϑ_j. We will generally suppose that preferences are homogeneous;

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[15]Maenhout (2004) reaches the same conclusion in an otherwise standard Merton model.

however, for reasons we outline in the next section, that will imply that ϑ will generally not be the same across countries.[16] To completely understand the mechanisms of the model, we will consider heterogeneity in ϑ directly without necessarily assuming heterogeneity in σ. Here, we use ϑ to denote the degree of robustness and use p to denote the resulting amount of model uncertainty.

In an integrated global equilibrium, there is a single risk-free interest rate i^*. Country j's expected growth rate can be written as

$$g^a = \psi_j \left(i^* - \delta_j \right) + \left(1 + \psi_j \right) \frac{\left(\mu^* - i^* \right)^2}{2 \widetilde{\gamma}_j \sigma^{*2}} \tag{24}$$

if the economy is governed by the approximating model, and

$$g^d = \psi_j \left(i^* - \delta_j \right) + \left(1 + \psi_j - \frac{2 \vartheta_j}{\gamma_j + \vartheta_j} \right) \frac{\left(\mu^* - i^* \right)^2}{2 \widetilde{\gamma}_j \sigma^{*2}} \tag{25}$$

if the economy is governed by the distorted model, where $\widetilde{\gamma}_j = \gamma_j + \vartheta_j$. If there exists a positive level of risk-free capital, we have $i^* = r$; if not, $i^* = \mu^* - \gamma^* \sigma^{*2} > r$.

Following Obstfeld (1994), the welfare gain from financial integration can be calculated as an equivalent variation: by what percentage must financial wealth be increased under financial autarky to leave the representative household indifferent to financial integration? Using (17), the equivalent variation for country j, Λ_j, can be written as:

$$\Lambda_j = \left(\frac{m_j^*}{m_j} \right)^{1/(1-\psi_j)} - 1 = \left(\frac{2 \psi_j \delta_j + \left(1 - \psi_j \right) \left(g_j^* + i^* \right)}{2 \psi_j \delta_j + \left(1 - \psi_j \right) \left(g_j + i_j \right)} \right)^{1/(1-\psi_j)} - 1, \tag{26}$$

where m_j and m_j^* are the marginal propensity to consume before and after financial integration. Another approach to compute the welfare gains is to follow Lucas' (1987) elimination-of-risk method. In Lucas (1987), the welfare cost of volatility is expressed in terms of percentage of consumption the representative agent in the endowment economy is willing to give up at all dates to switch to the deterministic world. However, such a welfare measure may not be very informative when the agent can make optimal consumption-portfolio choices because the marginal propensity of consumption is now endogenous and affects the trend of consumption growth. We thus follow Obstfeld (1994) to compute the welfare gains of risk sharing in this paper.

From the expression of the welfare gain, Λ_j, we can see that both an increase in growth and an increase in risk-free interest rate contribute to the welfare gain. In addition, the growth improvement can be different in an economy governed by the approximating model and an economy governed by the distorted model. Depending on the original growth rate before the financial

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[16]We have studied the effect of heterogeneous ϑ in our other work, in particular Luo, Nie, and Young (2012).

integration and how much growth is improved after the financial integration, an economy could experience larger welfare gains under the distorted model.

In general, we can prove that if the economy remains in a diversified equilibrium after integration, the welfare gain is larger if the economy is governed by the approximating model than if it is governed by the distorted model.

Proposition 4 *Let $\psi < 1$. The welfare gain from financial integration, measured by Λ_j, declines with the degree of robustness in the diversified equilibrium. In addition, for the same country, the welfare gain from financial integration under the distorted model is smaller than that under the approximating model.*

Proof. See Online Appendix B. ∎

It is worth noting that if the economy switches from a diversified equilibrium to an undiversified equilibrium after financial integration, the relative size of the welfare gain depends on the actual change in growth rates.

4   Quantitative Analysis

In this section, we calibrate the model, taking into account the key facts we want to explain. In particular, we first document the relevant facts on the correlation between economic growth and growth volatility across countries. Using more recent data, we confirm previous studies' findings that this relationship is generally negative if we pool all countries together, similar to Ramey and Ramey (1995), but the relationship is positive if we restrict the sample to be advanced countries and more financially integrated countries, similar to KPT (2005). We then utilize the analytical solution in our model to jointly calibrate the key parameters to match their empirical counterparts, including the estimated relationship between growth and volatility.

Regarding the key parameter on the degree of robustness, as a double check, we also follow the literature to calculate the corresponding detection error of probability (DEP) at the parameter values we calibrate for different groups of countries. As DEP provides an intuitive explanation on the amount of model uncertainty, our approach is not only consistent with the cross-country evidence (similar to the indirect reference approach) but also provides an alternative explanation based on a statistical tool.

Section 4.1 presents empirical evidence on the correlation between growth and volatility in developing and advanced countries. Section 4.2 discusses how to jointly calibrate the key parameters. Section 4.3 quantitatively computes and compares growth and welfare gains due to financial integration between developing and developed countries.

4.1 Empirical Evidence

In this section, we present evidence on the relationships across countries. We use both simple scatterplots and a regression analysis controlling the across-country heterogeneity to show these relationships.[17]

The data we use come from the Penn World Tables (version 8.0), which contains national accounts data on a wide set of countries that has been chained and converted to $USD. This conversion allows us to compare GDP levels between countries as well as across time. To choose which countries we include in our analysis, we begin by constructing a sample as close as possible to Ramey and Ramey (1995) but extend the time horizon to cover more recent years. In particular, our sample consists of 80 countries and covers the $1962 - 2011$ period.[18] Next, we classify the countries according to the groupings in Kose, Prasad, and Terrones (2006): industrial, more financially integrated (MFI), and less financially integrated (LFI).[19] This process delivers a final sample of 85 countries - 24 industrial, 21 MFI, and 40 LFI. The full list of countries can be found in Figure 1.

Figure 2 provides a graphical view of the relationships between GDP growth and its volatility by plotting the mean real per-capita GDP growth rate in the $1962 - 2011$ period for each country against its standard deviation. This does suggest a negative correlation between growth and volatility, which is consistent with the finding in Ramey and Ramey (1995). In addition, when we follow KPT (2006) to group countries into LFI, MFI, and advanced categories, this relationship switches from an overall negative one in LFI and MFI countries (Figures 3 and 4) to a significantly positive one in advanced countries (Figure 5).

However, this simple correlation may be biased due to the heterogeneity across countries. To provide a more accurate measure of the volatility and to isolate the connection to growth, we follow Ramey and Ramey (1995) by controlling for country-specific effects. Our control variables include mean investment share of GDP for each country over the sample period, real per-capita GDP in 1962 (logged), and a human capital index, all data that are included in the Penn World

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[17]We are not the first to study this relationship: other papers include Hnatkovska and Loayza (2004); Kose, Prasad, and Terrones (2005, 2006); Imbs (2007); and Miranda-Pinto (2016).

[18]There are 167 countries in the Penn World Tables dataset, 134 of which contain all of the variables used in the analysis (most countries excluded were missing the human capital variable). Of these 134 countries, 98 contain all variables of interest for the full sample period of 1962-2011. We removed an additional 18 countries since they were not contained in the Ramey and Ramey (1995) analysis. This procedure leaves us with a sample of 85 countries with observations from 1962 to 2011, yielding a total of 4000 observations.

[19]There are 9 countries (Barbados, Botswana, Democratic Republic of the Congo, Cyprus, Malta, Mozambique, Syria, Taiwan, and Uganda) that are included in Ramey and Ramey's sample but not in Kose's. Additionally, Korea and Israel are classified as MFI under Kose's definition. The IMF, however, classifies these countries as developed (World Economic Outlook, April 2016), so we will reclassify them as industrial.

Tables. We conduct a two-stage regression. In the first stage, we regress real per-capita GDP growth on the set of control variables and compute the residuals, which represent the GDP growth uncorrelated with our explanatory series. We compute the standard deviation of these residuals for each country to provide a measure of the volatility of unexplained growth. In the second stage, we put the measured volatility back into the original regression with the control variables for the second stage of the regression. Motivated by the above evidence that LFI countries, MFI countries and advanced countries show different relationships between growth and volatility, we include two dummy variables (for MFI and advanced countries, respectively) to capture the differences. As shown in Table 1, the volatility of GDP growth has a significant negative effect on per-capita real GDP growth for LFI countries.[20] In particular, a 1 percentage point increase in the standard deviation of growth leads to a 0.12 percentage point *decline* in growth in LFI countries. In contrast, for MFI and advanced countries, a 1 percentage point increase in the standard deviation of growth leads to 0.06 (0.18 minus 0.12) and 0.13 (0.25 minus 0.12) percentage-point *increases* in growth, respectively. These results are broadly consistent with KPT (2006) except that our coefficients for these three groups are all statistically significant while their estimate for MFI countries is not (see Table 2 of KPT 2006). This is possibly due to the fact that our data set extends theirs by 11 years and thus our estimation benefits from the additional data.

In terms of control variables, consistent with some AK-style endogenous growth models, we do find that high investment countries grow faster.[21] In addition, initial GDP level is negatively correlated with GDP growth while the role of human capital in influencing growth is not statistically significant. As we noted above, from Equation (23), it is clear that the standard full-information Obstfeld model cannot generate the negative relationship between the volatility and mean growth rate of real GDP per capita we observed in the data unless the economy is in an undiversified equilibrium with an EIS larger than 1. As discussed before, there is some uncertainty regarding the value of the EIS, so it is not clear that the model is a good instrument for measuring the welfare gains from diversification as they depend critically on the relationship between growth and volatility. In the next subsections, we will explore how and to what extent taking model uncertainty and the worst-case scenario into account can help make the model fit the data better in this aspect.

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[20] See also Imbs (2007); Dabusinskas Kulikov, and Randveer (2012); Hnatkovska and Loayza (2004); and Miranda-Pinto (2016) for alternative approaches that also find a negative correlation between growth and volatility.

[21] See McGrattan (1998) for a discussion of how this observation provides support for AK-style endogenous growth models.

4.2 Calibrating the Key Parameters

To quantitatively evaluate the changes in growth and welfare associated with financial integration under model uncertainty, we need to use reasonable values of the model parameters. We divide parameters into two groups to estimate and calibrate separately. In the first group, we calibrate each of them to a particular moment or take a value from the existing literature. In the second group, we jointly calibrate them so that the model-predicted moments match their empirical counterparts.

Our first group of parameters consists of 4 parameters: the return to risk-free capital, r; the return to risky capital, μ; the utility discount rate, δ; and the standard deviation of the risky-capital return, σ. We use the same value of r as in Wang, Wang, and Yang (2016). We use the stock-market returns and volatility in different countries to measure μ and σ and set the value of δ to be a small positive value based on Zhuang *et al.* (2007).[22] These values are reported in Table 3.

The second group of parameters consists of 3 parameters: the robustness parameter (ϑ), the EIS parameter (ψ), and the risk aversion parameter (γ). These three parameters are more difficult to pin down separately because they do not have direct targets to match individually. Therefore, we conduct a joint estimation by choosing their values so the model-predicted output growth, standard deviation of growth, and the elasticity of growth to the variance of growthóthe three core moments match their empirical counterparts. Notice that the elasticity of growth to variance of growth is the regression coefficient reported in Table 2. The idea of matching the estimated elasticity is in line with the indirect reference approach. It also takes advantage of our analytical solutions that deliver an explicit expression of growth as a function of growth volatility. The calibration results, which show the model fits the key moments in the data perfectly, are reported in Table 4. Comparing the parameters for the three groups of countries, the EIS (ψ) is similar across different groups of countries, the degree of risk aversion (γ) is stronger in industrial countries and MFI countries than in LFI countries, and the robustness parameter (ϑ) is larger in LFI and MFI countries than in industrial countries.

The values of the RB parameter, ϑ, do not have an intuitive explanation without further guidance. For example, even though we can see ϑ is larger in developing countries than in advanced countries, what does that mean? How can we link the difference in parameter values in the two groups of countries to the difference in the amount of model uncertainty in the two groups? To answer these questions and to better interpret the key parameter, we adopt the

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[22]What matters for the calibration is the gap between the return to risk-free capital and the discount parameter, $r - \delta$. In order for the model to match growth in the data, δ needs to be small so this gap is larger than the average growth in different groups of countries.

procedure outlined in AHS (2003) and Hansen and Sargent (Chapter 9, 2007) to calculate the detection error probability (DEP) associated with the value of the RB parameter (ϑ).[23] The intuition is as follows. The DEP is a statistic concept to measure the difference between the two models (in our case, the approximating model and the distorted model). It tells us the probability that a likelihood ratio test cannot distinguish one model from the other model. In other words, a larger DEP means it is more difficult to distinguish two models. This also means the distance between the two models is smaller or the agent is taking into account a smaller range of models when making decisions. Similarly, a smaller DEP means the agent is taking into account a larger range of models in making decisions or the agent has stronger preference for robustness. Figure 9 illustrates how DEP (p) varies with the value of ϑ for different values of γ.[24] The figure shows that for given values of γ, the stronger the preference for robustness (higher ϑ), the less the value of p is. For example, when $\gamma = 3$ and $\psi = 0.63$, $p = 0.43$ when $\vartheta = 1$, while $p = 0.36$ when $\vartheta = 3$. Both values of p are reasonable as argued in AHS (2002) and Maenhout (2004).

Following this procedure, the corresponding DEPs for LFI, MFI, and industrial countries are 0.17, 0.18, and 0.38, respectively. Given the negative relationship between the degree of model uncertainty and DEP, this suggests the amount of model uncertainty in LFI countries is highest, followed by MFI countries, with industrial countries having the least. It is worth noting that the amount of model uncertainty, measured by the DEP, is jointly determined by a country's fundamentals (i.e., the returns of assets and volatility of risky assets) and the RB parameter, which explains why LFI countries have the largest DEP even though their RB parameter (ϑ) is slightly smaller than that in MFI countries.

4.3   Quantifying Growth and Welfare Gains

Using the estimated parameters for industrial, MFI, and LFI countries, we can quantitatively explore the effects of financial integration on growth and welfare under model uncertainty. We report the results in Table 5.

As Panel A of the table shows, both MFI and industrial countries experience an increase in economic growth after financial integration, while LFI countries experience a decrease in growth after financial integration. This also suggests that the magnitude of the growth improvement seems to increase with the degree of financial integration (if we consider industrial countries as the most financially integrated group). Our analytical solution helps to understand these results. Recall that the expression for the expected growth rate under the distorted model, (25), shows growth is determined by two terms. The first term on the right side captures the effect

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[23]The technical details are provided in Online Appendix C.

[24]Here we also set $r = 0.04$, $\mu = 0.082$, and $\sigma = 0.02$ as in the previous section.

of changes in the equilibrium risk-free rate, i^*, on growth, while the second term highlights the effects of changes in the volatility of the risky-capital return, σ^*, on growth. The bottom panel of Table 5 shows that the equilibrium risk-free rate remains unchanged at 4 percent after financial integration. But Panel F of the table shows that the volatility of growth increases in all groups after financial integration. As shown by (25), the increase in the volatility of GDP growth has different impacts on growth in different groups with higher volatility leading to lower growth in LFI countries and higher growth in MFI and industrial countries. Also, consistent with the data and reflected in our calibration exercise, the positive impact for the industrial countries is larger than that for the MFI countries. Together, these findings explain why financial integration leads to larger growth improvement in industrial countries than in MFI countries and leads to lower growth in LFI countries. It is worth noting that the increase in growth volatility after financial integration is not driven by larger volatility of risky assets as Panel E shows the world risky assets are less volatile than any of the risky assets in the three groups of countries before financial integration. Instead, the increase in growth volatility is driven by the increase in the share of risky assets as shown by Panel C. In other words, as financial integration lowers the volatility of risky assets, the demand for risky assets ("world mutual fund") increases.

To explain why the volatility of GDP growth has different impacts on growth in different groups of countries, our model highlights the role of model uncertainty faced by different countries. In particular, our calibrated model shows that a less financially integrated country faces larger model uncertainty; therefore, the impact of growth volatility on growth is either smaller or more negative. This provides one possible explanation for the previous finding (KPT 2006) that financial integration alters the relationship between growth and volatility. That is, during the process of financial integration, the amount of model uncertainty faced by a country may decline and therefore generate positive effects on growth.

Next, a comparison in welfare improvements for the three groups shows a similar pattern, as shown in Panel B in Table 5. The improvement in welfare in industrial countries is around 60 percent and much higher than the improvement in MFI countries, while in LFI countries welfare declines by 27 percent. It is easy to understand this difference by using our formula (26) that shows an increase in growth (g^*) will increase the welfare improvement. Since growth improvement is the largest in industrial countries, it is not surprising their welfare improvement is also the largest. The welfare gains reported in Table 5 are consistent with the estimates found in some recent studies. While some of these studies report small gains, a majority of them report significant welfare gains in both advanced and developing countries. For example, van Wincoop (1999) finds that the potential welfare gains from international risk sharing in the OECD countries

can be as high as 3.5 percent.[25] Athanasoulis and van Wincoop (2000) find that the welfare gain from risk sharing related to growth volatility is about 6.5 percent for a data set of 49 developed and developing countries. It is also worth noting that the welfare gains calculated from using equity returns are much larger than those calculated using international consumption data. For example, Lewis (2000) finds that the welfare gains from international risk sharing based on equity returns are around 10 to 50 percent. The large difference is mainly due to the high volatility of equity returns and the implied intertemporal substitution in the marginal utility.

To sum up, our calibration exercise based on three groups of countries suggests that financial integration benefits industrial countries more than developing countries with industrial countries experiencing larger growth and welfare improvement than MFI countries and LFI countries experiencing decline in growth and welfare after financial integration. However, in order to focus on the role of model uncertainty and financial integration on growth and welfare, we have made some important assumptions in the benchmark model. In particular, we have assumed all countries have access to the same risk-free assets before financial integration. As Caballero, Farhi, and Gourinchas (2017) show, the set of safe assets is very limited and most countries do not really own any safe assets. This explains why investors in less financially developed countries, such as China, want to purchase international safe assets and why we see "flight-to-safety" phenomenon driving down U.S. sovereign yields. In the following subsection, we provide alternative calibrations by allowing heterogeneity in risk-free assets.

4.4 Alternative Calibrations: Different r in LFI Countries

For simplicity, we only alter the assumption on LFI countries to show how heterogeneity in the risk-free rate r may change growth and welfare implications. In the alternative calibrations, we assume the true risk-free assets in LFI countries have a lower return than that in the industrial countries and MFI countries, and thus financial integration allows LFI countries to purchase the global risk-free assets that have a higher return. In order to still match the growth rate and growth volatility in LFI countries, we adjust the other two parameters, μ and δ. The results are reported in Table 6. The top panel of the table assumes the risk-free rate is 3.5 percent in LFI countries before financial integration while the bottom panel assumes an even lower risk-free rate, 3 percent. In both cases, financial integration allows LFI countries to have access to a higher return on risk-free assets.[26]

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[25]van Wincoop (1999) also shows that the computed welfare gains from better international insurance depend heavily on the underlying model economy.

[26]We assume the equilibrium risk-free rate is the maximum risk-free rate in the three groups of countries if the equilibrium is still a diversified one. If the equilibrium becomes an undiversified one, the equilibrium interest rate is determined the usual way (based on the formula of the share of risky assets).

Two new findings emerge in these alternative scenarios. First, the access to a higher risk-free rate leads to better growth and welfare outcomes for LFI countries. This can be seen by the smaller decrease (and even an increase) in growth and welfare. This improvement for LFI countries comes from the interest rate channel as shown in the first term of the growth formula (25). Second, the larger the gap in the return between the LFI's own risk-free assets and the equilibrium global risk-free assets, the more benefits LFI countries can gain from financial integration. For example, in Alternative 2 in which LFI countries' risk-free rate increases from 3 percent to 4 percent after financial integration, the associated welfare gain for LFI countries switches from negative to positive. The improvement in welfare for the LFI countries is again due to an increase in the risk-free rate that offsets the decrease in growth as can be seen using our welfare-change formula (26).

As our alternative calibration strategy leads to a different value of δ for LFI countries that influences the welfare comparison (see our formula (26)), in Table 6, we also report the associated welfare changes for LFI countries assuming δ is unchanged (labeled as "fixed δ").[27] The results show that they lead to smaller changes in welfare for LFI countries, but qualitatively the main conclusions do not change. Overall, this alternative calibration highlights the additional benefits for LFI countries from having access to higher return safe assets, which improves their growth and welfare.

5   Conclusion

The relationship between average growth and average volatility are heterogeneous for different groups of countries. For example, it is negative in developing countries but positive in advanced countries. We show introducing model uncertainty due to a preference for robustness into an otherwise standard Obstfeld (1994) model helps explain the observed heterogeneity in the relationship between the growth rate and the volatility of real GDP; in contrast, the basic model cannot replicate this relationship. Our calibrated model shows advanced countries benefit more than developing countries in financial integration, and the difference in the growth-volatility relationship is the key to explaining this result. Our model could be extended to include other sources of risk such as fiscal policy (Eaton 1981) and then to assess which countries gain from integration and how that is related to policy choices.

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[27]It is worth noting a different δ has no effect on growth comparison because $r - \delta$ is unchanged in the alternative calibration strategy, which is the term through which δ influences growth.

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Industrial	MFI	LFI	
Australia	Argentina	Bangladesh	Nepal
Austria	Brazil	Barbados	Niger
Belgium	Chile	Bolivia	Panama
Canada	China	Botswana	Paraguay
Cyprus	Colombia	Burundi	Senegal
Denmark	Egypt	Cameroon	Sierra Leone
Finland	Hong Kong	Congo, Dem. Rep.	Sri Lanka
France	India	Costa Rica	Syria
Germany	Indonesia	Cote d`Ivoire	Tanzania
Greece	Jordan	Dominican Republic	Togo
Ireland	Malaysia	Ecuador	Trinidad & Tobago
Israel	Mexico	El Salvador	Tunisia
Italy	Morocco	Fiji	Uganda
Japan	Pakistan	Gabon	Uruguay
Korea, Republic of	Peru	Ghana	Zambia
Netherlands	Philippines	Guatemala	Zimbabwe
New Zealand	Singapore	Honduras	
Norway	South Africa	Iran	
Portugal	Thailand	Jamaica	
Spain	Turkey	Kenya	
Sweden	Venezuela	Lesotho	
Switzerland		Malawi	
Taiwan		Mauritius	
United Kingdom		Mozambique	

Figure 1: List of Countries

All Countries

Figure 2: Growth and Volatility of Per-Capita GDP (1962-2011): All Countries

Std Dev of Per Capita GDP Gro
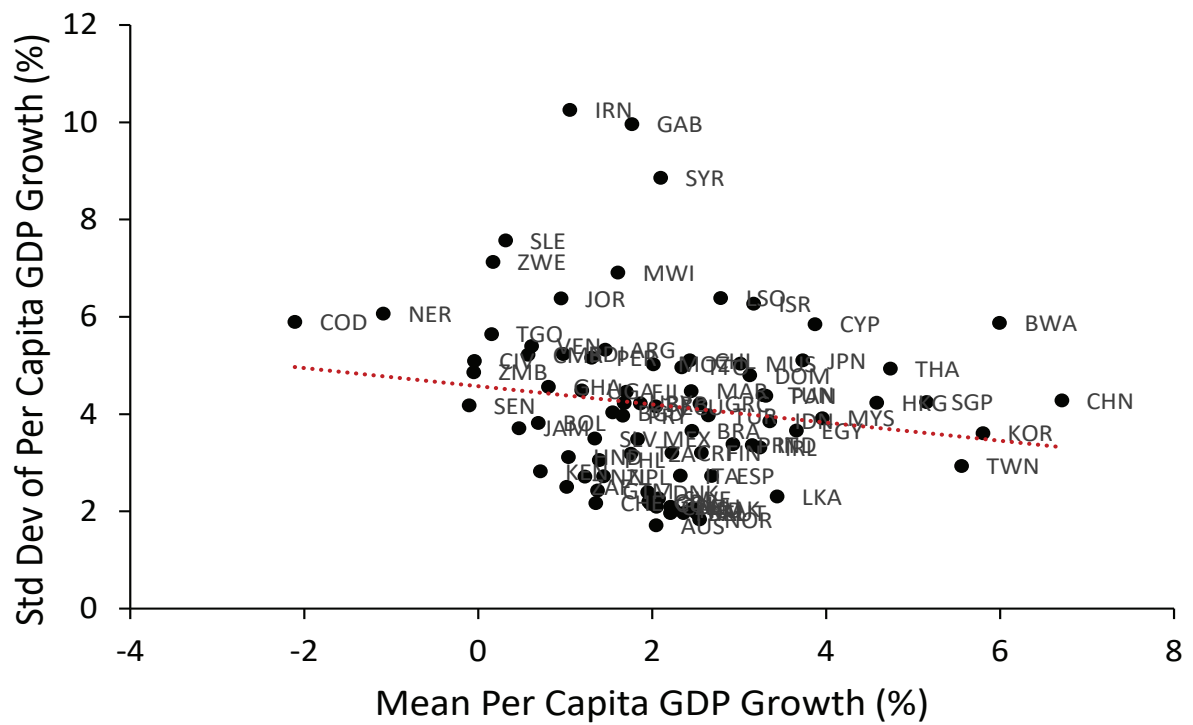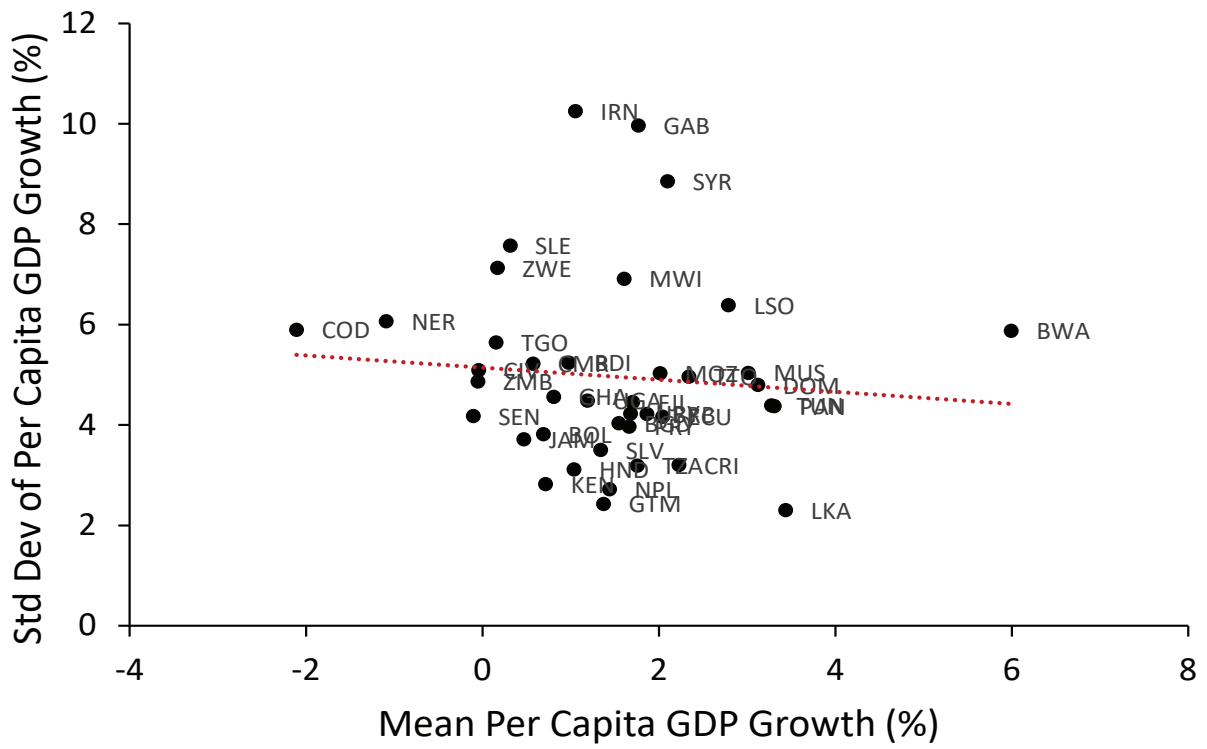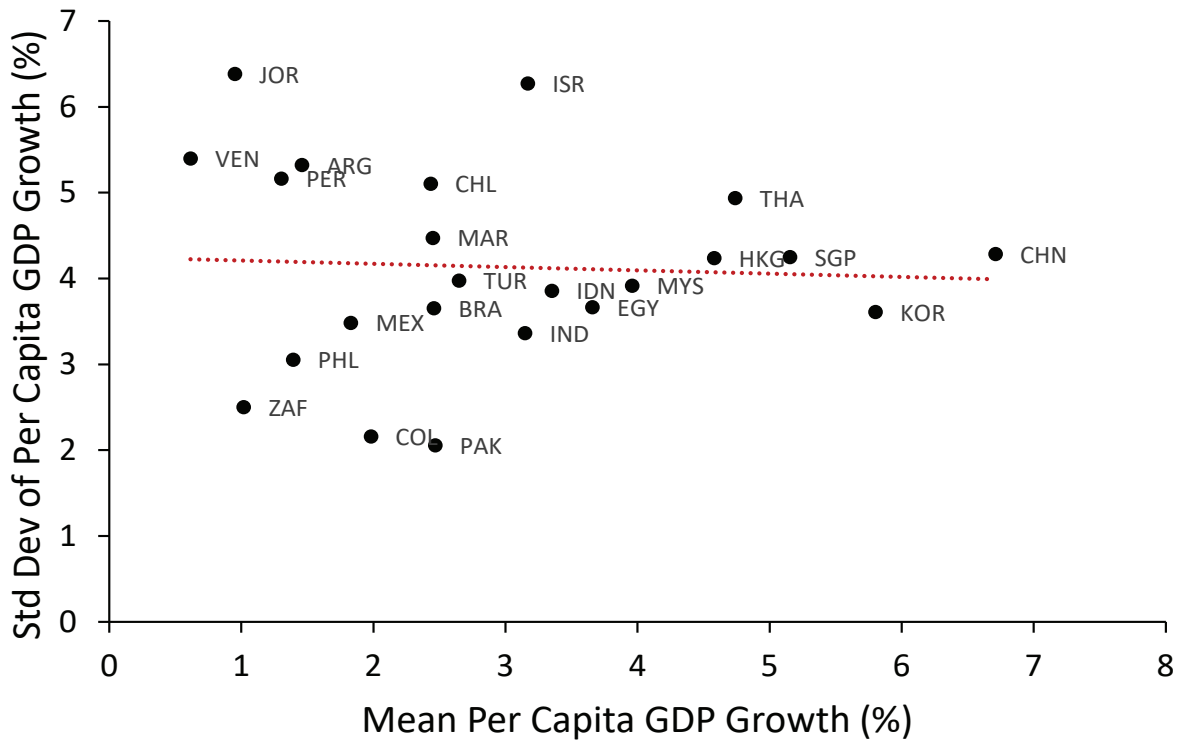

Figure 3: Growth and Volatility of Per-Capita GDP (1962-2011): LFI Countries

Figure 4: Growth and Volatility of Per-Capita GDP (1962-2011): MFI Countries

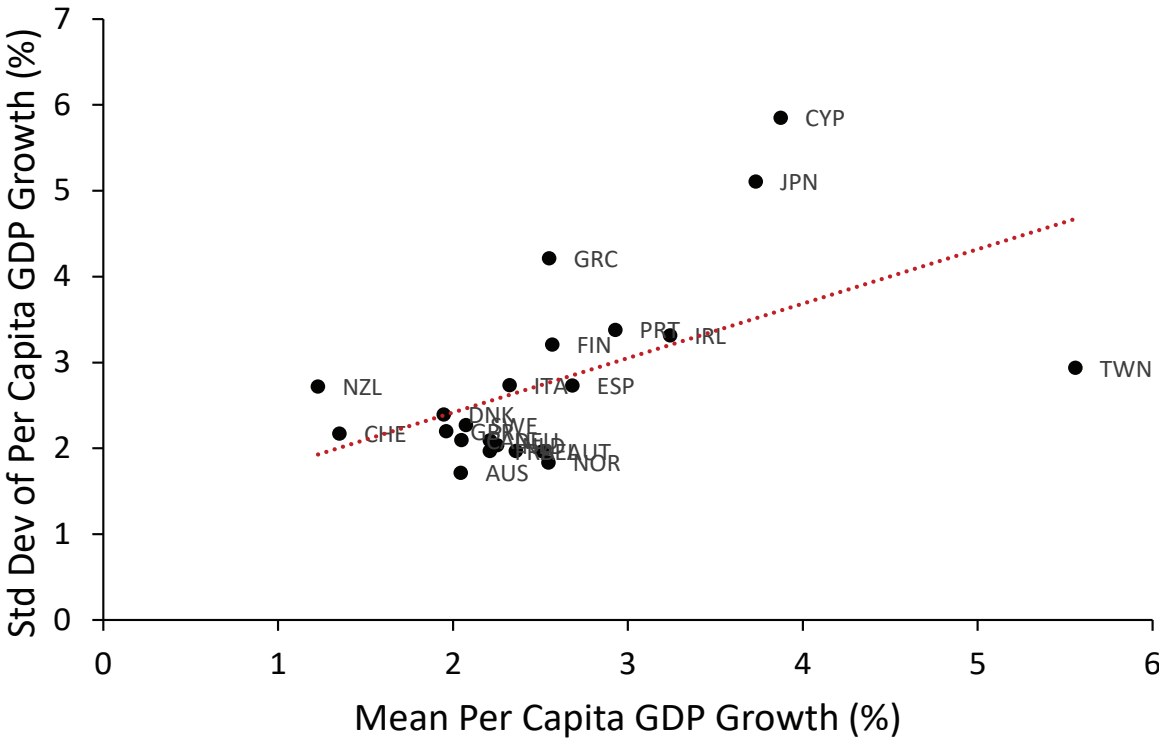Std Dev of Per Capita GDP Gro

Industrial Countries

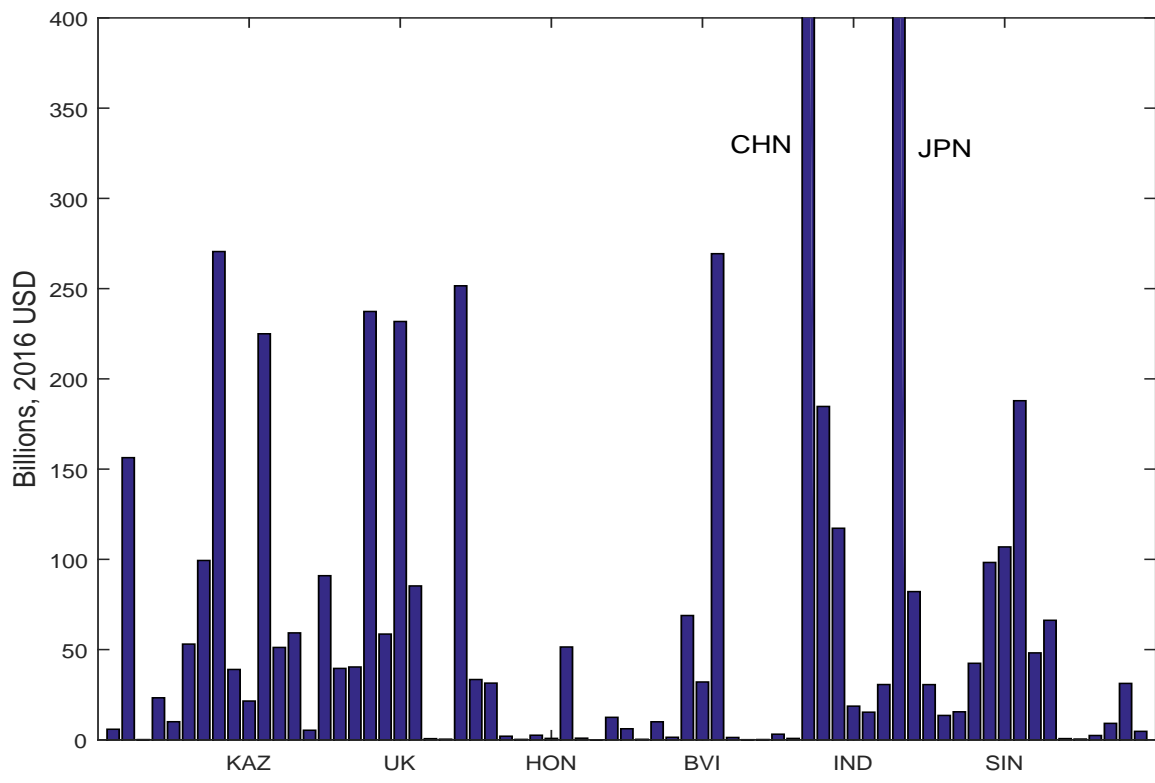Figure 5: Growth and Volatility of Per-Capita GDP (1962-2011): Industrial Countries

Figure 6: Distribution of US Treasury Debt
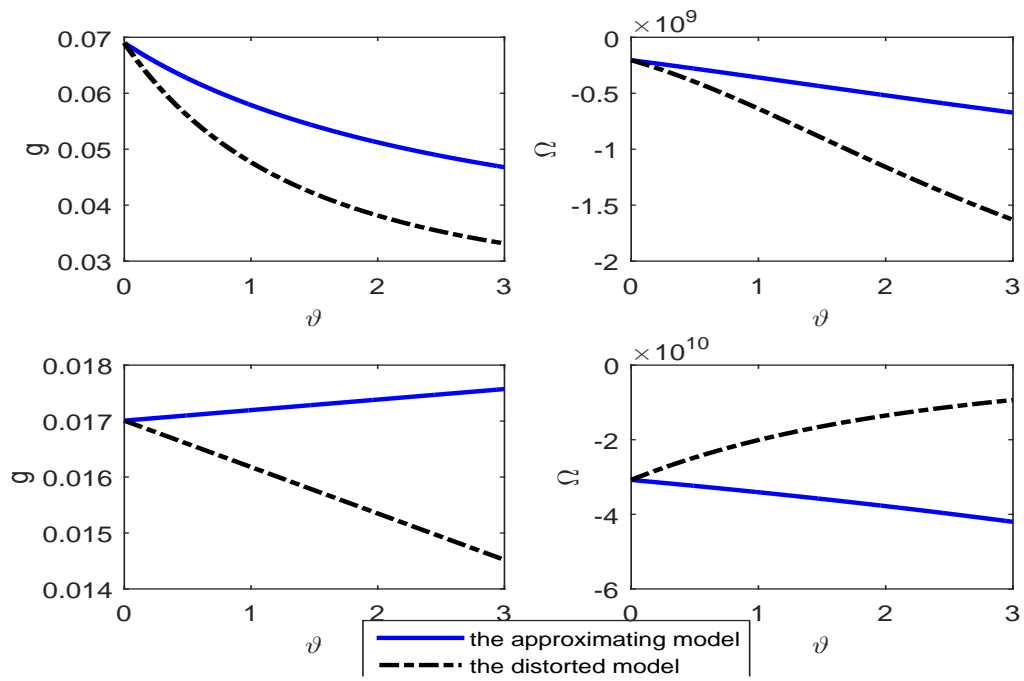
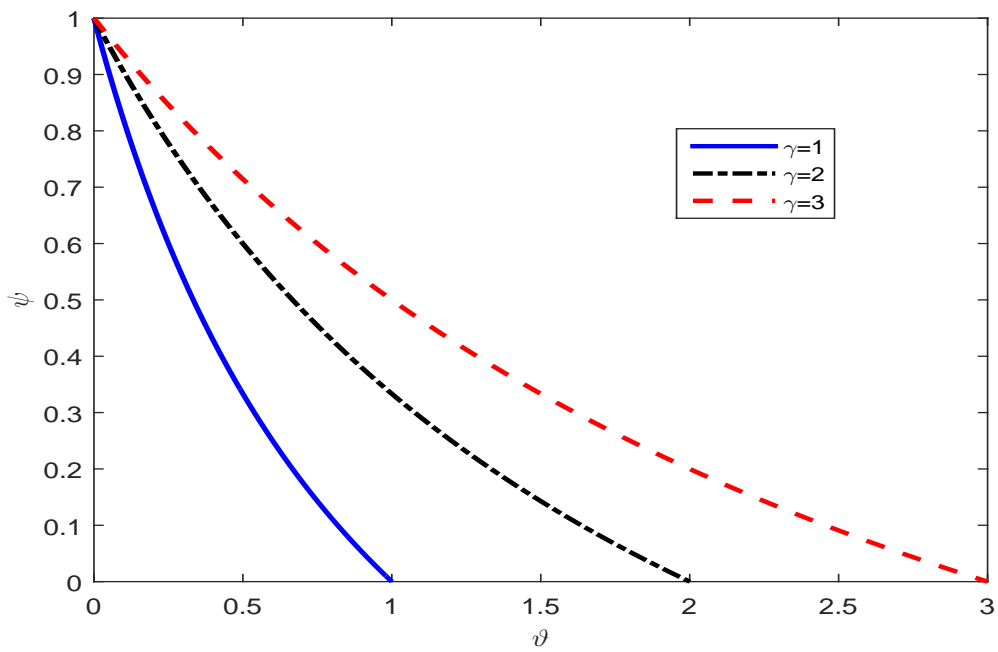Figure 7: Effect of RB on Growth in the Diversified Equilibrium
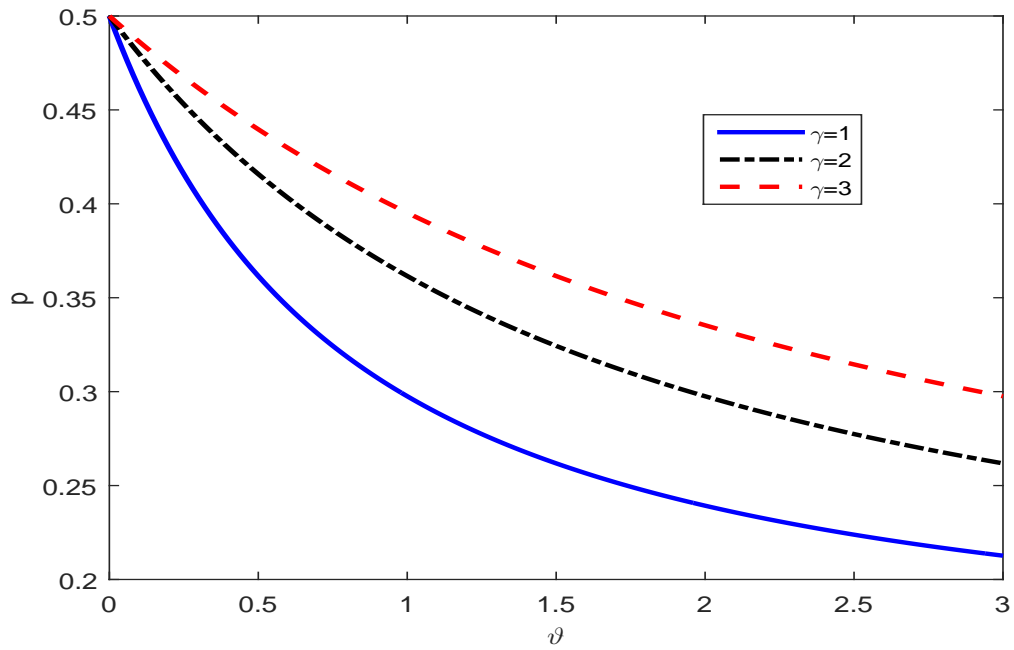

Figure 8: Relation between EIS and RB

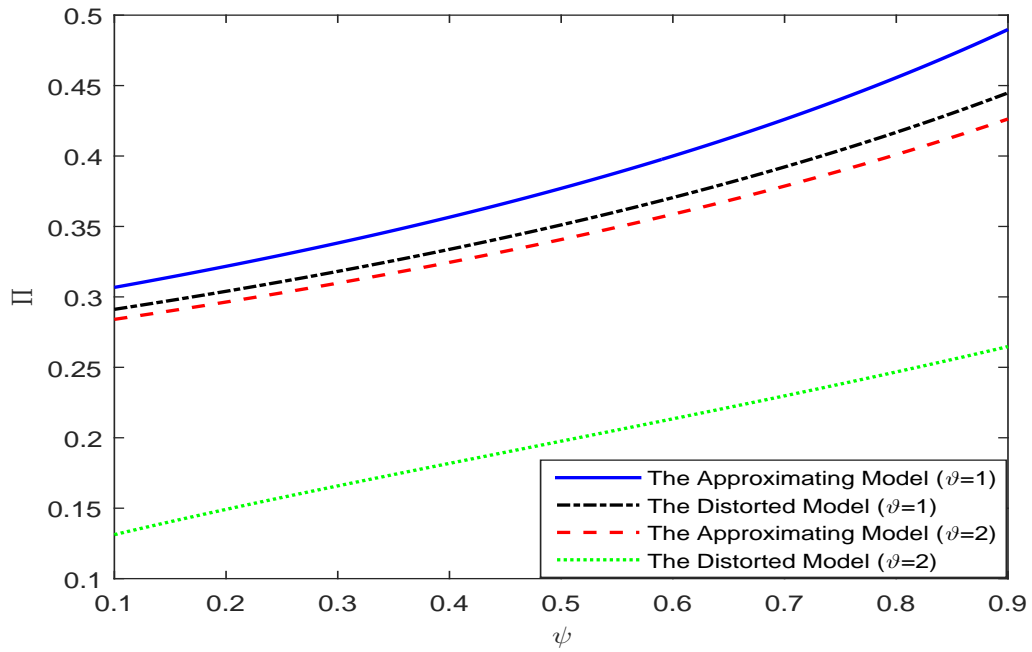Figure 9: Relationship between ϑ and p

Figure 10: The Welfare Gain from Intergration under RB

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Table 1: Regression Results on the Effects of Volatility on Per-Capita GDP Growth (1962-2011)

Independent Variables	Regression Coefficient	p-value
Volatility, $std(g)$	-0.124	0.004
MFI Dummy, $I_{MFI} \cdot std(g)$	0.178	0.000
Industrial Dummy, $I_{IND} \cdot std(g)$	0.246	0.000
Investment Share	0.109	0.000
Human Capital	-0.001	0.704
Initial Per-Capital GDP Level	-0.005	0.000
Constant	0.038	0.000

Table 2: Regression Results on the Effects of Σ^2 on g^d (1962-2011)

Independent Variables	Regression Coefficient	p-value
Variance, Σ^2	-1.279	0.002
MFI Dummy, $I_{MFI} \cdot \Sigma^2$	2.545	0.004
Industrial Dummy, $I_{IND} \cdot \Sigma^2$	3.646	0.004
Investment Share	0.114	0.000
Human Capital	0.000	0.775
Initial Per-Capital GDP Level	-0.004	0.000
Constant	0.029	0.000

Table 3: Values of Group 1 Parameters

Parameters		LFI	MFI	Industrial	Target
return to risk-free capital	r	0.04	0.04	0.04	Wang et al. (2016)
discount parameter	δ	0.001	0.001	0.001	Zhuang et al. (2007)
return to risky assets	μ	0.113	0.169	0.082	stock-market data
volatility of risky assets	σ	0.262	0.339	0.209	stock-market data

Table 4: Estimation of Group 2 Parameters

Parameters and Targets	LFI Countries	MFI Countries	Industrial Countries
Parameters			
ψ	0.47	0.65	0.63
γ	0.84	2.81	3.53
ϑ	5.08	6.14	2.82
DEP	0.17	0.18	0.38
Data Moments			
Growth (%)	1.45	2.78	2.71
Std. of Growth	5.37	4.25	3.19
Reg. Coef.	-1.28	1.27	2.37
Model-Generated Moments			
Growth (g)	1.45	2.78	2.71
Std (g)	5.37	4.25	3.19
Reg. Coef.	-1.28	1.27	2.37

Table 5: Growth and Welfare Gains from Financial Integration under RB

	Before Integration	After Integration	Percent Change
Panel A: Growth Rate (%)			
LFI	1.45	0.57	−60.9%
MFI	2.78	2.97	6.7%
Industrial	2.71	4.02	48.2%
Panel B: Welfare (Before = 100)			
LFI	100	72.6	−27.4%
MFI	100	107.8	7.8%
Industrial	100	160.0	60.0%
Panel C: Share of risky invest. (α)			
LFI	0.153	0.608	
MFI	0.205	0.352	
Industrial	0.125	0.497	
Panel D: Return of risky-capital return (μ)			
LFI	0.113	0.123	
MFI	0.169	0.123	
Industrial	0.082	0.123	
Panel E: Volatility of risky-capital return (σ)			
LFI	0.262	0.163	
MFI	0.339	0.163	
Industrial	0.209	0.163	
Panel F: Volatility of growth (Σ)			
LFI	0.054	0.099	
MFI	0.043	0.057	
Industrial	0.032	0.081	
Panel G: Risk-free Rate (i)	4.0%	4.0%	

Table 6: Growth and Welfare Gains under Alternative Assumptions for LFI Countries

	Before Integration	After Integration	Percent Change
Alternative 1: Risk-free Rate for LFI = 3.5%			
Growth Rate (%)			
LFI	1.45	0.85	−41.4%
MFI	2.78	2.95	6.2%
Industrial	2.71	3.96	46.0%
Welfare (Before = 100)			
LFI	100	95.6	−4.4%
LFI (fixed δ)	100	96.3	−3.7%
MFI	100	107.1	7.1%
Industrial	100	155.8	55.8%
Alternative 2: Risk-free Rate for LFI = 3%			
Growth Rate (%)			
LFI	1.45	1.13	−22.2%
MFI	2.78	2.94	5.6%
Industrial	2.71	3.90	44.0%
Welfare (Before = 100)			
LFI	100	148.7	48.7%
LFI (fixed δ)	100	129.2	29.2%
MFI	100	106.4	6.4%
Industrial	100	153.0	53.0%