Highlights

Inflation's Role in Optimal Monetary-Fiscal Policy*

Eric M. Leeper, Xuan Zhou

Highlight 1 Jointly optimal monetary and fiscal policy with commitment ascribes to inflation a significant role in financing shocks that increase fiscal needs.

Highlight 2 The importance of innovations in current and expected inflation as marginal sources of optimal financing increases with the maturity and the level of outstanding government debt.

Highlight 3 At debt levels and maturity lengths advanced economies now and expect to experience, as much as 50 percent of an innovation to fiscal stress would optimally be financed by current and expected inflation.

Highlight 4 Maturity structure can attenuate the distortions that sticky prices create.

Inflation's Role in Optimal Monetary-Fiscal Policy

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Abstract

We address the optimal marginal source of financing shocks that raise fiscal needs in the presence of a maturity structure for nominal government debt, distortionary taxes, and sticky prices. We find: (1) the importance of innovations in current and expected inflation that revalue debt increases with both the average maturity and the level of debt; (2) an analytical trade off between inflation and output-gap stabilization as a function of debt maturity; (3) at current debt levels and maturity lengths in advanced economies, inflation would account for as much as 50 percent of marginal optimal financing; (4) maturity attenuates welfare losses from nominal rigidities.

Keywords: inflation, tax smoothing, debt management, debt maturity, fiscal finance
JEL Codes: E31, E52, E62, E63

1. Introduction

Advanced economies have been hit recently by two unusually powerful shocks—the global financial crisis and the COVID-19 pandemic—with correspondingly large fiscal repercussions. Table 1 records that public debt jumped after each shock and, if the pattern following the financial crisis is repeated, is likely to remain elevated at 2021 levels for years to come. Optimal fiscal finance is now a pressing policy issue, just as it was after World War II.

	2007	2012	2019	2021
Canada	22.9	35.8	23.4	37.0
Euro Area	52.0	73.4	69.2	82.8
Japan	80.5	135.4	150.4	172.3
United Kingdom	38.0	83.7	75.3	97.2
United States	48.2	83.8	83.0	109.0

Table 1: General government net debt-to-GDP percentages. 2021 data are IMF projections. Source: International Monetary Fund (2012, 2020).

This paper addresses the optimal *marginal* source of fiscal financing. Tax revenues, surprise current inflation, and surprise changes in bond prices—which reflect innovations in expected inflation—can all serve as marginal sources of financing. What is the optimal mix of financing following a shock that affects the government's fiscal needs? How does the optimal mix change with the level and average maturity of government debt?

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An influential line of work ascribes to inflation only a minimal role in optimal fiscal finance [Schmitt-Grohé and Uribe (2004), Siu (2004), and Faraglia et al. (2013)]. Schmitt-Grohé and Uribe (2004, p. 200) put a sharp point on their finding: "...a miniscule amount of price stickiness suffices to bring the optimal degree of inflation volatility close to zero," leaving taxes to fully finance government spending disturbances, as in Barro (1979) and Aiyagari et al. (2002). We label as "conventional wisdom" the view that optimal policy ascribes a trivial role to inflation in fiscal financing.

Empirical evidence, though, finds that surprise changes in inflation and bond prices, which revalue debt, are not small potatoes. Hall and Sargent's (2011) ex post fiscal accounting finds that about 20 percent of the decline in the U.S. debt-GDP ratio from 1945 to 1974 came from negative real returns due to unanticipated inflation. Sims (2013) estimates surprise capital gains and losses on federal debt to be the same order of magnitude as fluctuations in primary surpluses. Bianchi and Ilut (2017) attribute much of the decline in U.S. debt in the 1970s, as well as the increase in inflation, to a policy mix that employed innovations in current and expected inflation to finance fiscal expenditures. That policy experience, Bianchi and Melosi (2017) argue, created uncertainty about how the expansion in debt from the 2008 financial crisis would be financed: by placing positive probability on inflation financing, agents' beliefs about future policy prevented the deflation that might otherwise accompany the effective lower bound on the nominal interest rate. Leeper, Traum, and Walker's (2017) ex ante accounting estimates that surprises in current and expected inflation account for between 7 and 25 percent of the financing of U.S. federal government spending shocks, depending on sample period and prevailing monetary-fiscal regime. Hilscher et al. (2018) argue that with sufficiently long maturity and a reasonable probability of persistently high inflation, inflation could substantially lower the U.S. fiscal burden. Cochrane (2020b) finds that half of the variation in the market value of debt is due to discount factor variation, with inflation accounting for 40 percent of movements in those factors.

The maturity structure of debt underlies all the empirical evidence. Table 2 reports the average term-to-maturity of outstanding government debt in some advanced economies in 2020. Across OECD members, the weighted average is almost 8 years, up from 6.2 years in 2007. These observed maturity lengths substantially alter the nature of optimal monetary and fiscal policies.

Country	Years
Canada	5.7
Japan	8.9
South Korea	11.0
United Kingdom	17.8
United States	5.4
OECD average	7.7

Table 2: Average term-to-maturity of outstanding marketable debt, 2020. Source: OECD (2021).

This paper aims to reconcile optimal policy with the empirical evidence that inflation contributes *significantly* to fiscal financing. We follow the approach that Benigno and Woodford (2004, 2007) develop to obtain analytical results that sharply characterize the policy trade offs, which existing work largely studies numerically. Deviations from optimality stem from distorting taxes, nominal price rigidities, and a distorted steady state. We derive optimal settings of the one-period nominal interest rate and the tax rate under full commitment and a timeless perspective, taking the maturity structure of debt as given. Shocks to fiscal needs arise from the impacts of underlying exogenous disturbances—technology, wage markups, government purchases, and transfer payments—on the government solvency condition.

Consistent with existing new Keynesian models, our calibrated micro-founded welfare function makes inflation volatility two orders of magnitude more costly than output gap volatility. Despite society's strong preference for stable inflation, longer-term debt gives inflation a significant role in optimal fiscal financing.

Key findings include:

1. There is always a role for innovations in current and expected inflation (through bond prices) to revalue debt, a role that increases with debt maturity and the level of debt.

2. At the elevated debt levels in table 1, inflation optimally absorbs significant fractions—over 50 percent—of shocks to fiscal needs with debt of maturity lengths like those of the United Kingdom in table 2.

3. As average maturity varies from one-period to infinity, optimal outcomes evolve from smoothing the expected price level to smoothing the expected output-gap.

4. Given actual U.S. debt levels and maturities, optimal policy would have used inflation to finance 14 percent of a shock to fiscal needs following World War II. After the financial crisis, if the maturity were extended, inflation's share would rise to 40 percent.

5. Debt maturity attenuates the welfare losses from nominal rigidities over an empirically relevant range of degrees of price stickiness.

Analytical and numerical results from a conventional model calibrated to postwar U.S. data deliver these findings.

1.1. Contacts with Literature

Schmitt-Grohé and Uribe's (2004) conventional wisdom that inflation should play only a minimal role in financing contrasts with Phelps's (1973) classic argument that in a second-best world, optimal policy balances distortions among various taxes, including using inflation to tax service flows and returns from nominal assets. That argument finds voice in neoclassical models with flexible wages and prices. Chari et al. (1996) and Chari and Kehoe (1999) show that an optimal policy generates jumps in inflation that revalue nominal government debt, much as inflation does under the fiscal theory of the price level [Leeper (1991), Sims (1994), Woodford (1995)].

Sims (2001, 2013) questions whether the conventional wisdom is robust when governments issue long-term nominal debt. He lays out an argument for surprise revaluations of nominal debt as a cushion against fiscal shocks to substitute for large movements in distorting taxes. Sims stops short of claiming that the weak responses of taxes to fiscal disturbances, which long debt permits, is optimal policy. Lustig et al. (2008) and Faraglia et al. (2013) show that it is optimal for the government to use more inflation to hedge against fiscal shocks with long-term debt. But Faraglia et al. conclude that price stickiness still dominates the desire to use inflation to stabilize debt, so inflation's role remains minor.

Existing efforts to quantify the importance of inflation in fiscal financing apply different methods designed to answer different questions. Hall and Sargent (2011), Lustig et al. (2008), and Faraglia et al. (2013), for example, perform backward-looking accounting exercises to address: "How have realized inflation rates contributed to the evolution of government debt?" We ask a different question focused on marginal sources of financing: "How are shocks that increase fiscal needs optimally financed in the future?" Because the sources of financing—current inflation and bond prices, future tax revenues, and future real interest rates—are forward-looking, we decompose the marginal increase in fiscal needs into each of these components.

The decomposition includes a real interest rate component that consists of two opposing effects—"interest-rate twisting" and "discount factors"—driven by fluctuations in output gaps. Interest-rate twisting refers to using a higher real interest rate to reduce the market value of debt. Discount factors directly affect the value of a given stream of primary surpluses that back debt. Faraglia et al. (2013) analyze twisting, but neglect discount factor impacts. Cochrane (2020b) considers only discount factors, estimating that they account for half the variation in the value of debt. We examine the two effects jointly to find that discount factor effects always dominate. We also find that the net effect of the two real interest rate effects converges to zero as maturity extends.

Analytical results clarify the trade offs between optimal inflation and output-gap stabilization. Debt maturity plays two distinct roles: smoothing the variables that determine welfare and allowing inflation to cushion against shocks to fiscal needs. Longer maturity enhances smoothing of both variables. But longer maturity makes it desirable to have inflation react more strongly and the output gap less strongly to shocks. Extreme maturities illuminate the cushioning role. With only one-period debt, the case that underlies conventional wisdom, optimal policy makes the price level a martingale and adjusts taxes to force the output gap to absorb disturbances. When nominal government bonds are consols, optimal outcomes flip: the output gap is a martingale and inflation adjusts permanently to exogenous shocks.

We identify a novel relationship between debt maturity and price stickiness and emphasize two aspects of the relationship. First, maturity structure upends the conventional wisdom that a little bit of stickiness eliminates the desirability of using inflation to stabilize debt. With even modest average maturity, it is optimal to rely heavily on surprise changes in bond prices. Innovations in bond prices smooth inflation over the maturity length of outstanding debt to produce large revaluations of debt with moderate inflation volatility. Second, maturity can attenuate the welfare losses from nominal rigidities. When prices are slightly or extremely sticky, maturity contributes little to welfare. At empirically relevant degrees of stickiness, maturity structure can offset welfare losses from sticky prices.

2. Model

Consider an economy that consists of a representative household with an infinite planning horizon, a collection of monopolistically competitive firms that produce differentiated goods and face nominal rigidities, and a government. A fiscal authority finances exogenous expenditures with distorting taxes and debt with a given maturity structure; a monetary authority sets the short-term nominal interest rate.

2.1. Households

The economy is populated by a continuum of identical households. Each household has preferences defined over consumption, C_t, and hours worked, N_{jt}. Preferences are

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_{jt}) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{jt}^{1+\varphi}}{1+\varphi} dj \right]$$

where σ^{-1} is the intertemporal elasticity of substitution, and φ^{-1} is the Frisch elasticity of labor supply.

Household consumption is a CES aggregator defined over a basket of goods of measure one and indexed by j, $C_t = \left[\int_0^1 C_{jt}^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$, where C_{jt} represents the quantity of good j consumed by the household in period t. The parameter $\epsilon > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods. Each good j is produced using industry-specific labor, with N_{jt} the quantity of labor supply of type j in period t. The household supplies all types of labor.

The aggregate price index P_t is $P_t = \left[\int_0^1 P_{jt}^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$, with P_{jt} is the nominal price of the final goods produced in industry j.

Households maximize expected utility subject to the budget constraint

$$C_t + Q_t^M \frac{B_t^M}{P_t} + \frac{1}{i_t} \frac{B_t^S}{P_t} = (1 + \rho Q_t^M) \frac{B_{t-1}^M}{P_t} + \frac{B_{t-1}^S}{P_t} + \int_0^1 \left(\frac{W_{jt} N_{jt}}{P_t \mu_t^W} + \Pi_{jt} \right) dj + Z_t$$

where W_{jt} is the nominal wage rate in industry j, Π_{jt} is the share of profits paid by the jth industry to the households, Z_t is lump-sum government transfer payments, and $\mu_t^W \geq 1$ is an exogenous wage markup factor common to all labor markets.[3] B_t^M is a government debt portfolio with price Q_t^M. The portfolio consists of perpetuities with coupons that decay exponentially [Eusepi and Preston (2018); Woodford (2001)]. A bond issued at date t pays ρ^{k-1} dollars at date $t+k$, for $k \geq 1$ and $\rho \in [0,1]$ is the coupon decay factor that parameterizes the *average maturity* of the debt portfolio. A consol emerges when $\rho = 1$ and one-period debt arises when $\rho = 0$. The duration of the long-term debt portfolio B_t^M is $(1-\beta\rho)^{-1}$. We impose that one-period bonds, B_t^S, are in zero net supply.

2.2. Firms

A continuum of monopolistically competitive firms produce differentiated goods. Production of good j is $Y_{jt} = A_t N_{jt}$ where A_t is an exogenous aggregate technology shock, common across firms. Firm j faces the demand schedule $Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon} Y_t$. With demand imperfectly price-elastic, each firm has some market power, leading to the monopolistic competition distortion in the economy.

Another distortion stems from nominal rigidities. Prices are staggered, as in Calvo (1983), with a fraction $1-\theta$ of firms permitted to choose a new price, P_t^*, each period, while the remaining firms cannot

[3]We follow Benigno and Woodford (2007) to include this "pure" cost-push shock. Chari et al. (2009) show that there are many underlying models that yield an equivalent gap between the real wage and the marginal rate of substitution. We can think of households as wage setters and firms as wage takers, with inefficient contracting between them.

adjust their prices. Firms that can reset their price choose P_t^* to maximize expected discounted profits, $\max E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}[(1-\tau_{t+k})P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})]$ subject to the demand schedule $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$, where $Q_{t,t+k}$ is the household's stochastic discount factor for the price at t of one unit of composite consumption goods at $t+k$, defined by $Q_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$. Sales revenues are taxed at rate τ_t, Ψ_t is cost function, and $Y_{t+k|t}$ is output in period $t+k$ for a firm that last reset its price in period t.

The first-order condition for this maximization problem implies that the newly chosen price in period t, P_t^*, satisfies $(P_t^*/P_t)^{1+\epsilon\varphi} = (\epsilon/(\epsilon-1))(K_t/J_t)$, where K_t and J_t are aggregate variables defined in Online Appendix B.

2.3. Government

The government consists of monetary and fiscal authorities who face the consolidated budget identity

$$\frac{Q_t^M B_t^M}{P_t} + \frac{1}{i_t}\frac{B_t^S}{P_t} + S_t = \frac{(1+\rho Q_t^M)B_{t-1}^M}{P_t} + \frac{B_{t-1}^S}{P_t}$$

where S_t is the real primary budget surplus defined as $S_t = \tau_t Y_t - Z_t - G_t$. G_t is government demand for the composite good and Z_t is government transfer payments. We consider a fiscal regime in which both G_t and Z_t are exogenous processes and only τ_t adjusts endogenously to ensure government solvency.

An intertemporal equilibrium—or solvency—condition links the real market value of outstanding government debt to the expected present value of primary surpluses. After imposing the expectations theory of the term stucture, that condition is

$$\left[1 + E_t \sum_{k=0}^{\infty} \frac{\rho^{k+1}}{i_t i_{t+1} ... i_{t+k}}\right] \frac{B_{t-1}^M}{P_t} = E_t \sum_{k=0}^{\infty} R_{t,t+k} S_{t+k}$$

where $R_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}}$ is the k-period real discount factor.

The price level today must be consistent with expected future monetary and fiscal policies, whether those policies are set optimally or not. Debt maturity introduces a fresh channel for expected monetary policy—choices of short-term nominal interest rates, i_{t+k}—to affect the current price level through the government's solvency condition.

2.4. Equilibrium

Goods market clearing requires $Y_t = C_t + G_t$ and labor market clearing requires $\Delta_t^{\frac{1}{1+\varphi}} Y_t = A_t N_t$. $\Delta_t = \int_0^1 (\frac{P_{jt}}{P_t})^{-\epsilon(1+\varphi)} dj$ measures price dispersion across firms, the source of welfare losses from inflation variability, which satisfies the recursive relation

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right]^{\frac{\epsilon(1+\varphi)}{\epsilon-1}} + \theta\pi_t^{\epsilon(1+\varphi)}\Delta_{t-1}$$

3. The Optimal Policy

In the optimal policy problem, government chooses functions for the tax rate, τ_t, and the short-term nominal interest rate, i_t, taking exogenous processes for technology, A_t, the wage markup, μ_t^W, government purchases, G_t, and transfers, Z_t, as given. The optimal Ramsey problem chooses optimal paths for $\{Y_t$, π_t, τ_t, b_t, Δ_t, J_t, K_t, Q_t^M, $i_t\}$ to maximize the welfare of households given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, \xi_t)$$

subject to households and firms optimality conditions, the aggregate resource constraint, the law of motion for price dispersion, and the government's budget identity.[4] We stack the exogenous shocks into the vector $\xi_t = \{A_t, \mu_t^W, G_t, Z_t\}$.

In the unconstrained Ramsey problem, the presence of expectations of variables in the constraint set makes the optimal policy time inconsistent. To be comparable to existing work, we avoid the time-inconsistency problem by adopting Woodford's (2003) "timeless perspective." We formulate the Ramsey problem recursively as if the optimal rule had been computed in the remote past. Policy commits to a time-invariant policy rule that is optimal subject to an initial pre-commitment, with the property of self-consistency.

3.1. Linear-Quadratic Approximation

We compute a linear-quadratic approximation to the nonlinear optimal solutions, using the methods that Benigno and Woodford (2004) develop. Distorting taxes and monopolistic competition in product and labor markets conspire to make the deterministic steady state inefficient. With a distorted steady state, an *ad hoc* linear-quadratic representation of the problem does not yield an accurate approximation of the optimal policy.[5] The main issue arises from the presence of a linear term in the second-order approximation to the welfare loss function. In this case, a first-order approximation to the equilibrium conditions ignores second-order terms potentially relevant to welfare.[6]

We adopt Benigno and Woodford (2004)'s approach because it leads to neat analytical solutions that separate the channels through which long-term debt affects optimal allocations and it nests conventional analyses of both optimal inflation-smoothing and optimal tax-smoothing, to connect the two literatures.

The representative household experiences welfare losses that, up to a second-order approximation, are proportional to

$$\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(q_\pi \hat{\pi}_t^2 + q_x \hat{x}_t^2 \right) \tag{1}$$

The relative weight on output stabilization, $\lambda \equiv \frac{q_x}{q_\pi}$, depends on model parameters and is defined in Online Appendix G. $\hat{x}_t$ denotes the welfare-relevant output gap, defined as the deviation between $\hat{Y}_t$ and its efficient level $\hat{Y}_t^e$, $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^e$. Efficient output, $\hat{Y}_t^e$, depends on the four fundamental shocks and is given by $\hat{Y}_t^e = q_A \hat{A}_t + q_G \hat{G}_t + q_Z \hat{Z}_t + q_W \hat{\mu}_t^W$.[7]

3.2. Linear Constraints

Constraints on the optimization problem come from log-linear approximations to the model equations. The first constraint is the Phillips curve, which we follow Benigno and Woodford (2004) to rewrite as

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \hat{x}_t + \kappa \psi (\hat{\tau}_t - \hat{\tau}_t^*) \tag{2}$$

where $\psi = \frac{w_\tau}{\varphi + \sigma s_c^{-1}}$, $\hat{\tau}_t^* \equiv -\frac{1}{\kappa \psi} u_t$, and u_t is a composite cost-push shock that depends on all four exogenous disturbances, as defined in Online Appendix G.

The household's Euler equation produces a second constraint. After imposing market clearing, we write it as

$$\hat{x}_t = E_t[\hat{x}_{t+1}] + \frac{s_c}{\sigma} E_t[\hat{\pi}_{t+1}] - \frac{s_c}{\sigma} \left(\hat{i}_t - \hat{i}_t^* \right) \tag{3}$$

where $\hat{i}_t^* \equiv \frac{\sigma}{s_c} v_t$ is the setting of the short-term nominal interest rate that exactly offsets the composite demand-side shock, v_t, which is defined in Online Appendix G.

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[4] Online Appendix B details the constraints, first-order conditions, and deterministic steady state.

[5] The size of the steady state distortion is measured by Φ, which drives a wedge between the marginal product of labor, and the marginal rate of substitution, $-\frac{U_n}{U_c} = (1 - \Phi) MPN$, where $\Phi = 1 - \frac{1-\bar{\tau}}{\bar{\mu}^W} \frac{\epsilon - 1}{\epsilon} > 0$, depends on the the steady state tax rate, steady state wage markup and the elasticity of substitution between differentiated goods.

[6] See Kim and Kim (2003), Galí (2008) and Woodford (2011) for further discussion.

[7] Online Appendix C to F contain detailed derivations; Online Appendix F also defines the parameters Γ, q_A, q_G, q_Z and q_W.

If policy makers faced only constraints (2) and (3), monetary and fiscal policies could stabilize inflation and output completely to achieve the first-best outcome, $\hat{\pi}_t = \hat{x}_t = 0$, by setting $\hat{\tau}_t = \hat{\tau}_t^*$ and $\hat{i}_t = \hat{i}_t^*$. To reach this first-best outcome, policy must have access to a non-distorting source of revenues or to state-contingent debt that can adjust to ensure that the government's solvency requirements do not impose additional restrictions on achievable outcomes.

When non-distorting revenues are not available, policy choice can, in effect, convert nominal debt into state-dependent real debt by issuing long-term nominal debt. Policies must be consistent with the flow government budget identity

$$\hat{b}_{t-1}^M + f_t = \beta \hat{b}_t^M + (1-\beta)\frac{\bar{\tau}}{s_d}(\hat{\tau}_t + \hat{x}_t) + \hat{\pi}_t + \beta(1-\rho)\hat{Q}_t^M \tag{4}$$

where $s_d \equiv \bar{S}/\bar{Y}$ is the steady-state surplus-output ratio. f_t, a composite fiscal shock that reflects all four exogenous disturbances to the government's budget identity, is defined in Online Appendix G.

Absence of arbitrage between short-term and long-term debt delivers the fourth constraint on the optimal policy program

$$\beta\rho E_t \hat{Q}_{t+1}^M = \hat{Q}_t^M + \hat{i}_t \tag{5}$$

Iterating on this and applying a terminal condition yields the term structure relation $\hat{Q}_t^M = -E_t \sum_{k=0}^{\infty}(\beta\rho)^k \hat{i}_{t+k}$. When $\rho > 0$, the price of long-term debt at t is determined by the entire path of future short-term nominal interest rates, making intertemporal smoothing possible.

Solving the budget identity forward and imposing transversality and the term structure relation yields an intertemporal version of the solvency condition

$$\begin{aligned}\hat{b}_{t-1}^M + F_t =& \hat{\pi}_t + \frac{\sigma}{s_c}\hat{x}_t + (1-\beta)E_t\sum_{k=0}^{\infty}\beta^k[b_\tau(\hat{\tau}_{t+k} - \hat{\tau}_{t+k}^*) + b_x \hat{x}_{t+k}] \\ &+ E_t\sum_{k=0}^{\infty}(\beta\rho)^{k+1}(\hat{i}_{t+k} - \hat{i}_{t+k}^*)\end{aligned} \tag{6}$$

The sum $\hat{b}_{t-1}^M + F_t$ summarizes the fiscal stress that prevents complete stabilization of inflation and the welfare-relevant output gap. Given the definitions of τ_t^* and i_t^*, F_t reflects fiscal stress stemming from three conceptually distinct but related sources: the composite fiscal shock, f_t, the composite cost-push shock, u_t (through τ_t^*), and the composite aggregate demand shock, v_t (through i_t^*). F_t and parameters b_τ and b_x are defined in Online Appendix G.

With F_t fluctuating exogenously, complete stabilization of inflation and output, which requires $\hat{\tau}_t = \hat{\tau}_t^*, \hat{i}_t = \hat{i}_t^*$, will not generally satisfy (6) and the government would be insolvent. The additional fiscal solvency constraint prevents policy from achieving the first-best allocation.

4. Optimal Policy Analytics

This section characterizes the nature of optimal policy behavior under sticky prices; the flexible price solution is relegated to Online Appendix H.

4.1. Sticky Prices

Under sticky prices, policy seeks paths for $\{\hat{\pi}_t, \hat{x}_t, \hat{\tau}_t, \hat{i}_t, \hat{b}_t^M, \hat{Q}_t^M\}$ that minimize (1) subject to (2)–(5).

To aid interpretation, we express the optimality conditions for inflation and the output gap in terms of the lagrange multipliers for the term structure, L_t^q, the flow government budget identity, L_t^b

$$q_\pi \hat{\pi}_t = -\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d}(L_t^b - L_{t-1}^b) - L_t^b + \frac{1}{\beta}L_{t-1}^q \tag{7}$$

$$q_x \hat{x}_t = (\psi^{-1} - 1)(1-\beta)\frac{\bar{\tau}}{s_d}L_t^b - \frac{\sigma}{s_c}L_t^q + \frac{\sigma}{s_c}\frac{1}{\beta}L_{t-1}^q \tag{8}$$

$$\beta(1-\rho)L_t^b = L_t^q - \rho L_{t-1}^q \tag{9}$$

$$E_t L_{t+1}^b = L_t^b \tag{10}$$

We solve for state-contingent paths of $\{\hat{\pi}_t, \hat{x}_t, \hat{i}_t, \hat{\tau}_t, \hat{b}_t^M, \hat{Q}_t^M, L_t^q, L_t^b\}$ that satisfy the constraints and optimality conditions.

These conditions make it clear that debt maturity and disturbances to the government budget affect inflation and the output gap. The shadow price of the government budget identity, L_t^b, follows a martingale, according to (10), a property that reflects intertemporal smoothing in fiscal financing.[8] L_t^b measures how binding the solvency constraint is on fiscal policy. The term structure multiplier, L_t^q, measures the tightness of the solvency constraint on monetary policy by linking L_t^q to a distributed lag of L_t^b with weights that decay with ρ, debt's duration

$$L_t^q = \beta(1-\rho)\sum_{k=0}^{\infty} \rho^k L_{t-k}^b \tag{11}$$

Maturity structure matters through its implications for fiscal financing. How much monetary policy is constrained by fiscal financing depends on the entire history of shadow prices of the government budget, L_{t-j}^b, and the degree of history dependence rises with the average maturity of government debt. Restricting attention to only one-period debt, $\rho = 0$, makes $L_t^q = \beta L_t^b$. This eliminates history dependence through the term structure of interest rates to render monetary and fiscal policies equally constrained by fiscal solvency.[9] At the opposite extreme, consols set $\rho = 1$, so $L_t^q \equiv 0$ and current monetary policy is not constrained, regardless of how binding the government's budget has been in the past.

4.2. Stabilizing Optimal Policies

We examine some special cases that sharply characterize the optimal equilibrium and the stabilization roles of fiscal and monetary policy.

4.2.1. Only One-Period Bonds

With only one-period debt, long-term and short-term interest rates are proportional and L_t^b and L_t^q covary perfectly. Expressions for inflation, (7), and the output gap, (8), become

$$q_\pi \hat{\pi}_t = -\left(\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d} + 1\right)(L_t^b - L_{t-1}^b) \tag{12}$$

$$q_x \hat{x}_t = \left[\left(\psi^{-1} - 1\right)(1-\beta)\frac{\bar{\tau}}{s_d} - \beta\frac{\sigma}{s_c}\right] L_t^b + \frac{\sigma}{s_c} L_{t-1}^b \tag{13}$$

Condition (12) implies that inflation is proportional to the forecast error in L_t^b. Because (10) requires there are no forecastable variations in L_t^b, the expectation of inflation is zero and the price level follows a martingale

$$E_t \hat{\pi}_{t+1} = 0 \quad \Rightarrow \quad E_t \hat{p}_{t+1} = \hat{p}_t \tag{14}$$

Condition (13) makes the output gap a weighted average of L_t^b and L_{t-1}^b. Taking expectations yields

$$E_t \hat{x}_{t+1} - \hat{x}_t = -\frac{1}{\lambda}\frac{\sigma}{s_c}\left(\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d} + 1\right)^{-1} \hat{\pi}_t \tag{15}$$

so the expected change in the output gap is proportional to current inflation. The optimal degree of output gap smoothing varies with λ, the relative weight on the output gap in the loss function. Larger λ delivers more output-gap smoothing. Flexible prices are a special case with $\lambda = \infty$. Under most calibrations, λ is quite small, to deliver little smoothing of output gap. But the martingale property of L_t^b implies smoothing of expected future output gaps after a one-time jump. Taking expectations of (15) yields

$$\hat{x}_t \neq E_t \hat{x}_{t+1} = E_t \hat{x}_{t+2} = ... = E_t \hat{x}_{t+k} = ... \tag{16}$$

[8] As Benigno and Woodford (2004) note, the unit root in dynamics that emerge from optimal policy prevents the deterministic solution from converging to the steady state.

[9] This is the exercise that finds that active monetary/passive fiscal policies yield highest welfare [Schmitt-Grohé and Uribe (2007) and Kirsanova and Wren-Lewis (2012)].

Taken together, (14) and (16) state that with only one-period debt, optimal policies smooth the price level and use fluctuations in the output gap to absorb innovations in fiscal conditions. The reason is apparent: without long-term debt, policy cannot smooth inflation across time, and surprise inflation—and the resulting price dispersion—is far more costly than variations in the output gap; it is optimal to minimize inflation variability and use output (and tax rates) as a shock absorber.

4.2.2. Only Consols

When the government issues only consols, fiscal stress that moves long rates need not change short rates contemporaneously. Inflation and the output gap are now

$$q_\pi \hat{\pi}_t = -\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d}\left(L_t^b - L_{t-1}^b\right) - L_t^b \tag{17}$$

$$q_x \hat{x}_t = \left(\psi^{-1} - 1\right)(1-\beta)\frac{\bar{\tau}}{s_d} L_t^b \tag{18}$$

Condition (18) makes the output gap proportional to L_t^b, so the gap inherits the martingale property of L_t^b

$$E_t \hat{x}_{t+1} = \hat{x}_t \tag{19}$$

Taking expectations of (17) and combining with (18), yields

$$E_t \hat{\pi}_{t+1} - \hat{\pi}_t = \frac{\lambda}{\kappa(1-\psi)}\left(\hat{x}_t - \hat{x}_{t-1}\right) \tag{20}$$

Condition (20) implies that the expected change in inflation is proportional to the change in $\hat{x}_t$. The degree of inflation smoothing varies inversely with ψ, the coefficient on the tax rate in the Phillips curve.

Combining (19) and (20), we draw opposite conclusions from the case of one-period debt. With consols, intertemporal smoothing of L_t^b smoothes the output gap; fluctuations in inflation absorb disturbances to fiscal needs. Now the bond price can absorb fiscal shocks: bad news about future surpluses can reduce the value of outstanding debt, leaving the real discount factor unaffected. A constant real discount factor smoothes the output gap, which explains the absence of forecastable variations in the output gap. Variations in the bond price correspond to adjustments in expected inflation. The longer the duration of debt—higher ρ—the less is the required change in bond prices and future inflation for a given change in the present value of surpluses. Although with consols it is optimal to allow surprise inflation to absorb shocks, the expectation of inflation is stabilized after a one-time jump: $\hat{\pi}_t \neq E_t\hat{\pi}_{t+1} = E_t\hat{\pi}_{t+2} = ... = E_t\hat{\pi}_{t+k} = \ldots$.

4.2.3. General Case

We briefly consider intermediate values for the average duration of debt, $0 < \rho < 1$. Rewrite (7) and (8) using the lag-operator, $\mathbb{L}^j x_t \equiv x_{t-j}$

$$q_\pi \hat{\pi}_t = -\frac{(1-\beta)\frac{\bar{\tau}}{s_d}}{\kappa\psi}(1-\mathbb{L})L_t^b - (1-\mathbb{L})(1-\rho\mathbb{L})^{-1}L_t^b \tag{21}$$

$$q_x \hat{x}_t = (\psi^{-1}-1)(1-\beta)\frac{\bar{\tau}}{s_d}L_t^b - \frac{\sigma\beta}{s_c}(1-\rho)(1-\beta^{-1}\mathbb{L})(1-\rho\mathbb{L})^{-1}L_t^b \tag{22}$$

The optimality condition for debt that requires L_t^b to be a martingale may be written as $(1-\mathbb{B})E_{t-1}L_t^b = 0$ where $\mathbb{B}$ is the backshift operator, defined as $\mathbb{B}^{-j}E_t\xi_t \equiv E_t\xi_{t+j}$.

Taking expectations of (21) and (22) and applying the backshift operator, we obtain general expressions for the k-step-ahead expectations of inflation and the output gap

$$E_t\hat{\pi}_{t+k} = \rho^k\hat{\pi}_t + \rho^k\alpha_\pi(L_t^b - L_{t-1}^b) \tag{23}$$

$$E_t\hat{x}_{t+k} = \rho^k\hat{x}_t + (1-\rho^k)\alpha_x L_t^b \tag{24}$$

where $\alpha_\pi = \frac{1-\beta}{\kappa\psi q_\pi}b_\tau$ and $\alpha_x = \frac{1-\beta}{q_x}(\frac{b_\tau}{\psi} - b_x)$.

Equations (23) and (24) neatly encapsulate the policy problem. The first terms on the right stem from the welfare improvements that arise from smoothing. That both terms involve ρ^k means that longer maturity debt helps to smooth expectations of both inflation and output. The second terms bring in the government solvency dimension of optimal policy through the Lagrange multipliers. They capture the trade off between relying on variations in inflation to hedge against fiscal stress and using variations in output to absorb shocks. Maturity has opposite effects on the two variables. As maturity lengthens, changes in government solvency affect future inflation more strongly, while the output gap becomes less responsive. As maturity extends, it is optimal to trade off inflation for output stabilization. For any maturities short of perpetuities, $0 \leq \rho < 1$, as the forecast horizon extends, $k \to \infty$, expected inflation converges to zero whereas the expected output gap converges to $\alpha_x L_t^b$. Inflation is anchored on zero, but the output gap's "anchor" varies with the state at t.

5. Calibration

We calibrate the model to U.S. data to obtain quantitative implications. Table 3 reports the calibration. We take the model's frequency to be quarterly and set some of our parameters according to the estimates of Rotemberg and Woodford (1997), including $\beta = 0.99$, which implies a steady state real interest rate of about 4 percent; $\theta = 0.66$, which implies an average price duration of three quarters; $\varphi = 0.47$, yielding a Frisch elasticity of 2.1; and $s_c^{-1}\sigma = 0.16$, implying $s_c\sigma^{-1}$—the elasticity of expected output growth with respect to the expected real return—is 6.25. We set $\epsilon = 5$, to give a degree of market power that results in prices being set 25 percent higher than marginal cost on average, as in Schmitt-Grohé and Uribe (2007). U.S. data from 1947Q1 to 2019Q3 underlie the values of steady state tax rate, $\tau = 0.21$, and steady state debt level, $s_b = 0.46 * 4$, which implies an annualized debt-GDP ratio of 46 percent. Given the assumed steady state gross price markup of 1.25 and the assumed steady state tax rate, an overall level of steady state distortions, $\Phi = 0.4$, corresponds to a steady state wage markup of $\mu^W = 1.05$. Since the steady state government budget identity implies $\bar{\tau} - s_g - s_z = \left(\beta^{-1} - 1\right) s_b$, if we assume zero government transfers in the steady state, then the government spending-output ratio is $s_g = 0.19$. We estimate autoregressive processes for A_t, G_t, τ_t, Z_t shocks.[10] Following Galí et al. (2007), the wage markup shock is calibrated as an AR(1) process with persistence of 0.95 and standard deviation of 0.054.

Parameter	Definition	Value
β	discount rate	0.99
σ	the inverse of elasticity of output with respect to real return	0.13
φ	the inverse of Frisch elasticity of labor supply	0.47
θ	the fraction of firms cannot adjust their prices	0.66
ϵ	intratemporal elasticity of substitution across consumption goods	5
ϕ	overall steady state distortion	0.4
s_c	steady state consumption to gdp ratio	0.81
s_g	steady state government spending-gdp ratio	0.19
s_b	steady state debt-gdp ratio	0.46×4
$\bar{\tau}$	steady state tax rate	0.21
ρ_a	autoregressive coefficient of tech shock	0.71
ρ_g	autoregressive coefficient of government spending shock	0.90
ρ_z	autoregressive coefficient of transfer payment shock	0.42
ρ_w	autoregressive coefficient of wage markup shock	0.95
σ_e^a	standard deviation of innovation to tech shock	0.008
σ_e^g	standard deviation of innovation to government spending shock	0.015
σ_e^z	standard deviation of innovation to transfer payment shock	0.046
σ_e^w	standard deviation of innovation to wage markup shock	0.054

Table 3: Calibration of model parameters to U.S. data, 1947Q1–2019Q3.

Two important parameters affect the trade off between the output gap and inflation. The slope of the Phillips curve, $\kappa = 0.0336$, is in line with McCallum and Nelson's (2004) suggested range of 0.01 to 0.05. The relative weight on output-gap stabilization, $\lambda = 0.0067$, is between the value used in Benigno and Woodford

[10]Online Appendix I provides details. Our estimates are consistent with the literature.

(2007) ($\lambda = 0.0024$), and the numbers used in Galí (2008) ($\lambda = 0.02$) and Walsh (2010) ($\lambda = 0.25$). Although our value of λ implies inflation stability receives 150 times the welfare weight as output-gap stability, we nonetheless find that inflation plays a significant role in fiscal financing.

6. Effects of Debt Maturity: Smoothing and Stabilization

To explore how debt maturity affects trade offs between inflation and output stabilization, we rewrite the government intertemporal equilibrium condition (6) in terms of only inflation and the output gap

$$\hat{b}^M_{t-1} + F_t = \hat{\pi}_t + (1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t + E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}$$
$$-(1-\beta)b_\tau\left(\frac{1}{\psi}-1\right)E_t\sum_{k=0}^{\infty}\beta^k\hat{x}_{t+k} + E_t\sum_{k=0}^{\infty}\left[(1-\beta)\beta^k-(1-\beta\rho)(\beta\rho)^k\right]\frac{\sigma}{s_c}(\hat{x}_t-\hat{x}_{t+k})$$

Given fiscal stress, $\hat{b}^M_{t-1} + F_t$, this condition completely summarizes feasible paths of current and expected inflation and output gaps. Interactions between the two policies determine the reliance on variations in output gaps versus inflation rates. To distinguish the average maturity structure's impact on inflation and the output gap, we consider two polar sub-optimal cases: (i) complete stabilization of the output gap, which relies only on inflation as a shock absorber; and (ii) complete stabilization of inflation, using only the output gap to absorb shocks, as well as the optimal allocation under joint optimal policy.

6.1. Inflation Smoothing

Complete output-gap stabilization sets $\hat{x}_t \equiv 0$ and uses current and future inflation to fully absorb innovations to $\hat{b}^M_{t-1} + F_t$. This polar case eliminates the effect of maturity on output smoothing to focus on how alternative maturity structures dynamically allocate inflation. The pure inflation-smoothing problem is to minimize $\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t q_\pi \hat{\pi}_t^2$ subject to

$$\hat{b}^M_{t-1} + F_t = \underbrace{\hat{\pi}_t}_{\text{current inflation}} + \underbrace{(1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t}_{\text{inflationary effect of tax policy}} + \underbrace{E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}}_{\text{long-term debt price}}$$

where we have separated fiscal financing into three distinct sources.

Complete stabilization of the output gap requires changes in fiscal stress to be financed by innovations in current and expected inflation. The inflationary effect of tax policy is absent in Cochrane (2001) and Sims (2013). Without steady-state tax rates, $b_\tau = 0$, and the suboptimal equilibrium implies $\hat{\pi}_t = (1 - \beta\rho^2)(\hat{b}^M_{t-1} + F_t)$ and $E_t\hat{\pi}_{t+k} = \rho^k\hat{\pi}_t$, for $k \geq 1$. Unexpected changes in fiscal stress must be accommodated entirely by surprise variations in current and future inflation. The response of current inflation is increasing in fiscal stress and decreasing in average maturity. Future inflation is expected to decay at rate ρ. Inflation follows a martingale, $E_t\hat{\pi}_{t+k} = \hat{\pi}_t$, as in Sims (2013), when $\rho = 1$.

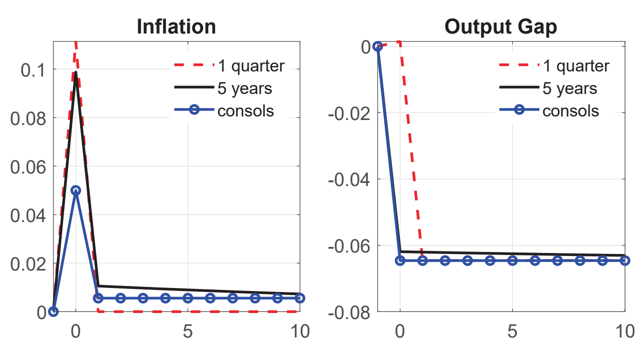

Figure 1: Response to a one-unit increase in fiscal stress of inflation when output gap completely stabilized (left panel) and of output gap when inflation completely stabilized (right panel).

Returning to our model with distorting taxes, figure 1 plots responses of inflation to a unit innovation in fiscal stress (left panel). With only one-period debt, inflation jumps immediately and then returns to zero, since intertemporal smoothing of inflation is unavailable. With five-year debt, inflation reacts less aggressively in the first period, and then gradually goes back to zero. Finally, with consols, the immediate response of inflation is the smallest, with future inflation permanently, but only slightly, higher. The presence of long-term debt allows the government to trade off inflation today for inflation in the future.

6.2. Output Gap Smoothing

Suppose $\hat{\pi}_t \equiv 0$ so that adjustments in current and future output gaps must absorb innovations to $\hat{b}^M_{t-1} + F_t$. Consider the pure output gap smoothing problem: minimize $\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t q_x \hat{x}_t^2$ subject to

$$\hat{b}^M_{t-1} + F_t = \underbrace{-(1-\beta)b_\tau \left(\frac{1}{\psi} - 1\right) E_t \sum_{k=0}^{\infty} \beta^k \hat{x}_{t+k}}_{\text{tax revenue}} + \underbrace{E_t \sum_{k=0}^{\infty} \left[(1-\beta\rho)(\beta\rho)^k - (1-\beta)\beta^k\right] \frac{\sigma}{s_c}(\hat{x}_{t+k} - \hat{x}_t)}_{\text{real interest rate}} \tag{25}$$

where again we separate fiscal financing into distinct bits.

Financing of enhanced fiscal stress occurs through three channels. First, higher tax rates raise revenues, which appear in (25) as lower output gaps. If this were the only channel, then the government's desire to smooth tax distortions shows up as smoothing the output gap, $E_t\hat{x}_{t+1} = \hat{x}_t$, as in Sims (2013).

The other two channels are folded into the real interest rate expression in (25). Variation in $\hat{x}_{t+k} - \hat{x}_t$ induces variation in the k-period real interest rate to create two opposing contributions to fiscal financing. A higher long-term real rate reduces the real market value of debt. Faraglia et al. (2013) call this "interest-rate twisting," and appears as $E_t \sum_{k=0}^{\infty}(1-\beta\rho)(\beta\rho)^k \frac{\sigma}{s_c}(\hat{x}_{t+k} - \hat{x}_t)$. But higher real rates also reduce discount factors, which reduce the present value of future tax revenues—the backing for debt. That component is $-E_t \sum_{k=0}^{\infty}(1-\beta)\beta^k \frac{\sigma}{s_c}(\hat{x}_{t+k} - \hat{x}_t)$. Cochrane (2020a) emphasizes the role of discount factors.[11]

Figure 1 plots responses of output gap to a unit innovation in fiscal stress (right panel). With only one-period debt, the output gap jumps on impact and then drops to a constant level, increasing real discount rates on tax revenues. With five-year debt, output drops in the first period and then gradually converges to the same level as with one-period debt, increasing real discount rates on tax revenues only slightly. At the maturity limit—consols—the output gap follows a martingale and real discount rates are unchanged.

6.3. Inflation and Output-Gap Stabilization

This section jointly determines output and inflation in the presence of distortionary taxes and sticky prices. Figure 2 plots responses of inflation and gaps under the fully optimal solution to a unit one-period increase in fiscal stress. Responses are a combination of the dynamic impacts in the two panels in figure 1. Both inflation and output adjust to absorb the disturbance in F_t. For all maturities, the output gap responds more aggressively than inflation. With only one-period debt, an increase in fiscal stress causes an immediate jump in inflation and a slight increase in output gap on impact; then the output gap drops to a permanently lower level, while inflation returns to its initial level. With five-year debt, output drops in the first period and then gradually converges to the same level as with one-period debt, while inflation reacts more aggressively than with one-period debt in the first period, and then gradually returns to zero. Because longer debt makes inflation smoothing available, it is less costly to use inflation to absorb shocks. With consols, the expected output gap is smoothed, while inflation jumps and then remains permanently higher.

Debt maturity structure alters the policy maker's constraint set to affect the feasible combinations of inflation and output-gap. Figure 3 shows the policy frontier by plotting the combination of optimal volatilities of inflation and output-gap with alternative values of $\lambda \in (0, \infty)$. We follow Woodford (2003) to measure the volatilities of inflation and output by the expected discounted value

[11] Linear utility or only consols make the net effect of the two interest-rate effects zero [Cochrane (2001) and Sims (2013)].

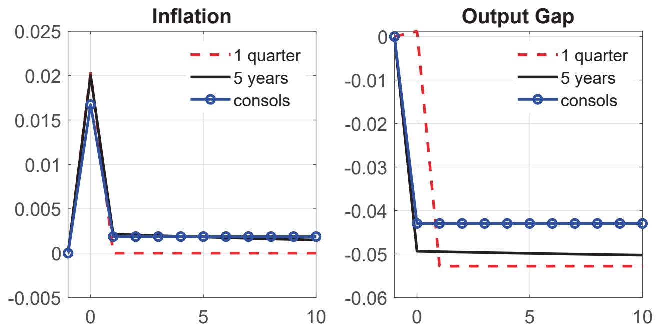

Figure 2: Inflation and output gap responses to a one-unit innovation in fiscal stress under alternative maturity structures.

$$V_\pi = (1-\beta)\sum_{t=0}^{\infty}\beta^t E_0 \hat{\pi}_t^2, \quad V_x = (1-\beta)\sum_{t=0}^{\infty}\beta^t E_0 \hat{x}_t^2$$

The policy frontier illustrates the trade-off between inflation and output-gap stabilization. Longer maturities relax the tensions between the two stabilization goals. When λ is small, stable inflation is more valued than stable output. For our calibration, $\lambda = 0.0067$, extending debt maturity increases the optimal variance of inflation only slightly, but decreases the optimal variance of the output gap significantly. Higher λ better stabilizes output. In our calibration, output is better stabilized than inflation ($V_x < V_\pi$) as long as $\lambda \geq 0.2$. As $\lambda \to \infty$, extending debt maturity significantly decreases the variability of inflation that must be accepted to completely stabilize the output gap.

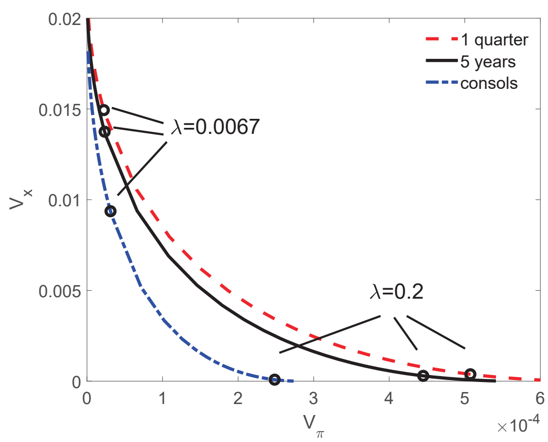

Figure 3: Policy frontiers for three different maturity structures. Circles mark optimal choices for two values λ, the weight on output relative to inflation stabilization.

7. Fiscal Financing

Sims (2013) emphasizes the role of surprise inflation as a fiscal cushion that can reduce reliance on distorting sources of revenues. One way to quantify the fiscal cushion is to use the government's solvency condition to account for the sources of fiscal financing—including current and expected inflation—following an innovation in the present value of fiscal stress, F_t. Write the government solvency condition as

$$\hat{b}^M_{t-1} + F_t = \underbrace{\hat{\pi}_t}_{\text{current inflation}} + \underbrace{E_t \sum_{k=1}^{\infty} (\beta\rho)^k \hat{\pi}_{t+k}}_{\text{expected inflation}} + \underbrace{(1-\beta)\left[\frac{b_\tau}{\kappa\psi}\hat{\pi}_t - b_\tau\left(\frac{1}{\psi}-1\right)E_t\sum_{k=0}^{\infty}\beta^k \hat{x}_{t+k}\right]}_{\text{tax revenue}}$$

$$+ \underbrace{\underbrace{E_t \sum_{k=0}^{\infty}(1-\beta\rho)(\beta\rho)^k \frac{\sigma}{s_c}(\hat{x}_{t+k}-\hat{x}_t)}_{\text{interest rate twisting}} + \underbrace{E_t \sum_{k=0}^{\infty}(1-\beta)\beta^k \frac{\sigma}{s_c}(\hat{x}_t - \hat{x}_{t+k})}_{\text{discount factor}}}_{\text{net effect of real interest rate}} \tag{26}$$

The left panels of figure 4 illustrate how a one-unit increase in fiscal stress is financed through current and expected inflation, tax revenue, and real interest rates, as a function of debt duration for the calibration to U.S. data in table 3 (solid lines). The vast majority of financing comes from an increase in tax revenues, but higher inflation and lower real interest rates also contribute to fiscal financing. As debt duration increases, innovations in expected inflation adjust more to absorb a shock to fiscal stress, while taxes adjust less. With longer duration debt, $\rho \to 1$, the importance of real interest rate adjustments dissipates. Because real interest rates transmit immediately into movements in the output gap, with short-duration debt distortions in output are relatively big. As duration rises, it is optimal to smooth output more, reducing real interest rate movements. In the limit, when $\rho = 1$, the net effect of real interest rates is zero and it is optimal to make $E_t\hat{x}_{t+1} = \hat{x}_t$ and to rely instead on inflation as a fiscal cushion, as expression (26) makes clear.

Our results differ from the literature along several dimensions. First, we adopt a forward- looking government solvency condition to account for the sources of fiscal financing. Hall and Sargent (2011) and Faraglia et al. (2013) adopt a backward-looking decomposition, which examines a historical interval and does not take into account household expectations of future polices: the effects of inflation cannot be isolated from those of interest rates, tax revenues and outstanding debt. Because we take future inflation into consideration, inflation generally plays a bigger role in our analysis.

Second, our decomposition does not depend on the specific shocks that increase fiscal stress: a unit increase in stress—from whatever exogenous source—always yields the same decomposition. Third, the right panels of the figure separate the response of real interest rates into the two opposing components—"interest rate twisting" and "discount factor"—expressed in (26). Quantitatively, the government's incentive to use a higher real interest rate to reduce the real market value of outstanding debt is dominated by its preferences for a higher discount factor to increase the present value of future tax revenues.

Sources of fiscal financing are particularly sensitive to the level of debt in the economy. Figure 4 also reports fiscal financing decompositions under two higher steady state debt-GDP levels like those in table 1 (dashed and dotted-dashed lines). As the level of debt rises, reliance on tax financing declines. With very short debt duration, changes in real interest rates account for a higher fraction of financing in high-debt economies. Reliance on real rates declines rapidly as duration rises, with future inflation becoming increasingly important. With long enough debt, high-debt economies would finance over 60 percent of a fiscal stress innovation with current and expected inflation.

How relevant is inflation's role if we consider the maturities and debt-GDP ratios observed in data? Figure 5 plots the average maturity, in years, of the marketable Treasury debt held by the public along with the debt-GDP ratio from 1941 to 2018 (left panels). Given the pair of observations at each date, we compute the optimal fiscal financing decomposition and plot the contributions of inflation and tax revenue with solid lines in the lower graph on the left. During the late 1940s, debt maturity averaged seven years, and the debt-GDP ratio was over 90 percent; an optimal policy would have financed 14 percent of an innovation to fiscal stress with current and expected inflation. From 2010 to 2018, as debt grew from 35 to 76 percent of GDP, the average maturity of government debt increased from 4.8 years to 5.8 years, to make inflation's contribution to optimal financing 7 percent.

The right panels plot counterfactuals. If the government had issued long debt (consols), current and expected inflation would have contributed as much as 50 percent to financing in the late 1940s and 40 percent after the financial crisis. If the government had issued short debt (1 period), then current and expected inflation play essentially no role in financing.

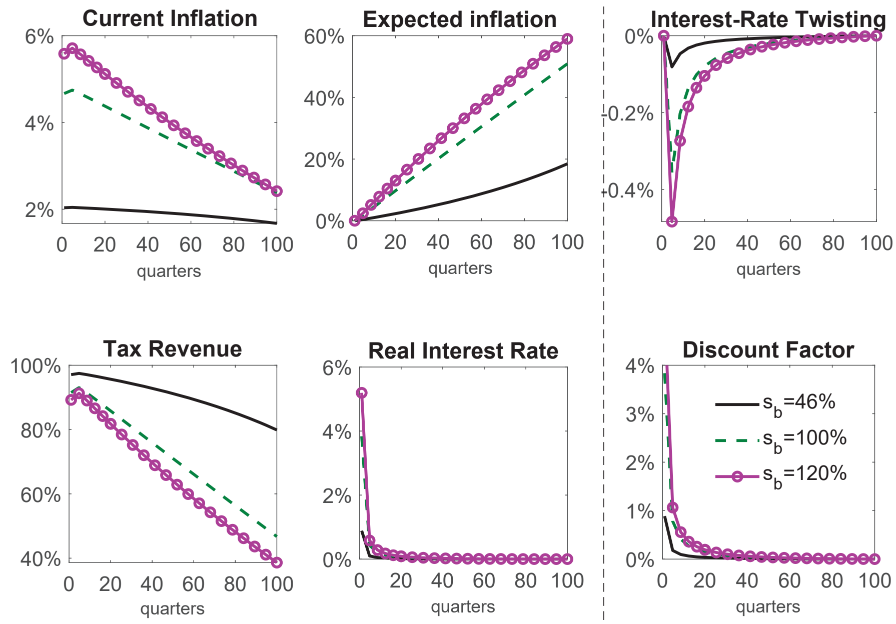

Figure 4: Left panel is fraction of fiscal stress innovation financed by each of the four components in (26), as a function of average duration of government debt. Right panel decomposes the net effect from real interest rates into its two components, as a function of average duration of government debt. Plotted for three steady state debt-output ratios, $s_b = 46\%, 100\%, 120\%$.

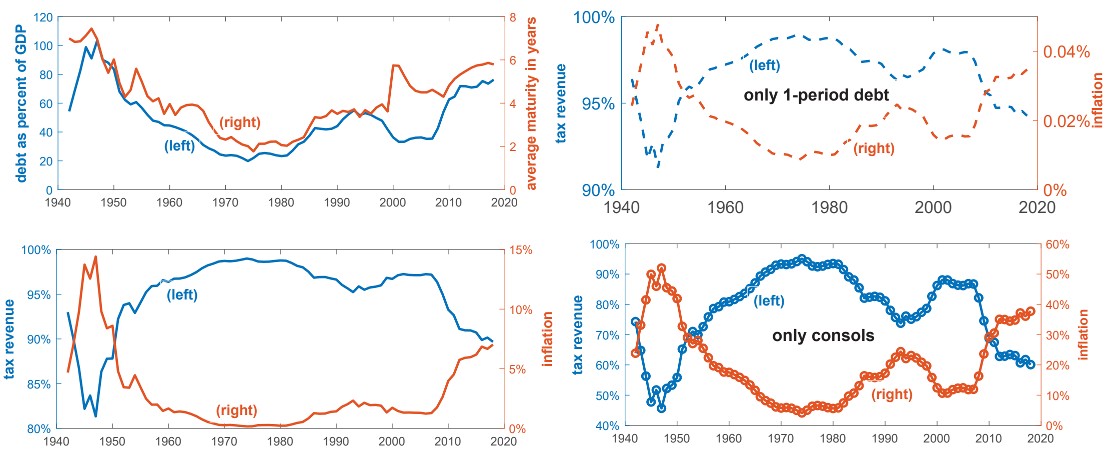

Figure 5: Inflation and tax revenue contributions to an innovation of fiscal stress given the debt maturity and debt-GDP ratio observed from 1940-2018 (left panels). Counterfactual financing given actual debt-GDP and two extreme maturity structures (right panels).

8. Debt Maturity and Price Stickiness

Maturity structure can attenuate the distortions that price stickiness creates. Maturity affects only the policy maker's constraint set by bringing expected inflation—and future monetary policy—into the government's solvency condition. Price stickiness, in contrast, affects both the constraint set—through the slope of the Phillips curve and the direct impact of tax rates on inflation in the solvency condition—and the policy maker's loss function—through the weight on inflation volatility. This second entrance of price stickiness in the optimality problem means that maturity cannot always fully offset stickiness distortions.

Figure 6 shows that maturity structure upends the conventional wisdom that a little bit of stickiness eliminates the desirability of using expected inflation to stabilize debt [Schmitt-Grohé and Uribe (2004), Siu (2004), Benigno and Woodford (2007) and Faraglia et al. (2013)]. When all debt is one-period—dashed lines—and prices are flexible, it is optimal to exploit surprise inflation, but no expected inflation to help achieve solvency. Even a small degree of stickiness leads to a sharp reduction in the use of surprise inflation, which rapidly becomes very costly. As maturity extends—solid and dotted lines—there is increasingly less reliance on surprise current inflation and more reliance on innovations in expected inflation. Consol debt makes the case starkly: use expected inflation and tax revenues exclusively, as section 4.2.2 showed.

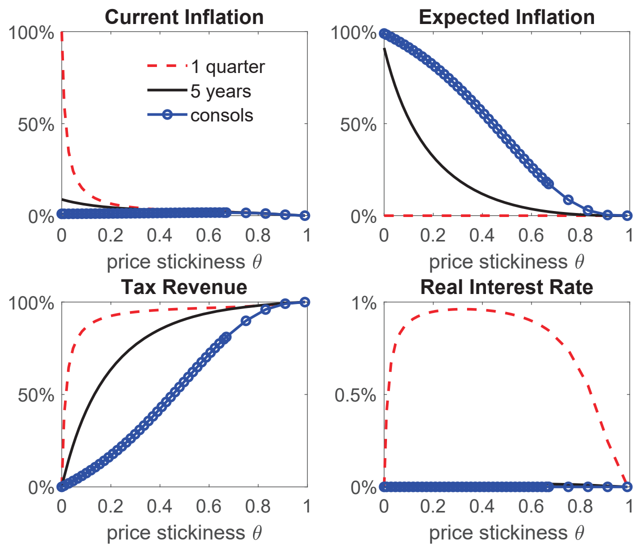

Figure 6: Fraction of fiscal stress innovation financed by each of the four components in (26), as a function of price stickiness, for three debt maturities

Figure 7 plots isoloss curves that show combinations of average maturity (y-axis) and stickiness (x-axis) that yield the same welfare loss for two steady-state debt-GDP levels. When prices are slightly or extremely sticky—low or high θ—maturity contributes little to welfare. At the moderate degrees of stickiness thought to be empirically relevant, maturity structure matters. Up to a point, welfare losses due to price stickiness can be offset by lengthening the maturity of outstanding debt. Beyond that point, if stickiness increases, debt maturity must decline to maintain welfare. Non-monotonicity arises from the dual roles that price stickiness plays in the policy problem, affecting both constraints and the loss function. Eventually, the degree of stickiness reaches a point where the welfare value of stabilizing current inflation dominates the value of stabilizing output, so maturity must shorten. Shorter maturity shifts expected inflation toward surprise current inflation.

When steady-state debt rises by magnitudes reported in table 1, isoloss curves shift appreciably. Consider a stickiness value of 0.6, which is in the ballpark of estimates. When debt is 46 percent of GDP, to achieve a loss of unity, average maturity must be 20 years; when debt is 100 percent, the same loss can be achieved with debt of only 6 years maturity. The figure underscores that for debt levels and maturities like those observed, maturity structure can make a valuable contribution to attenuating welfare losses that nominal rigidities create.

9. Contrast to Simple Rules

This section compares jointly optimal policies with two alternative policy regimes: inflation targeting monetary policy coupled with debt-stabilizing fiscal behavior or optimal policy subject to a balanced-budget rule. In the inflation targeting regime monetary policy obey the interest rate rule, $\ln(i_t/\bar{i}) = \alpha_\pi \ln(\pi_t/\bar{\pi})$, where i_t and π_t are levels, $\bar{i}$ and $\bar{\pi}$ are steady-state values; fiscal policy adjusts taxes in response to deviations of debt from steady state, $\ln(\tau_t/\bar{\tau}) = \gamma_b \ln(b_{t-1}/\bar{b})$. In the balanced budget regime, policies are chosen optimally but must satisfy the additional restriction that they balance the budget period by period.

9.1. How Big Is the Departure from Price Stability?

Figure 8 plots the volatilities of inflation and the output gap along with the welfare losses as a function of debt maturities for jointly optimal policy and two uncoordinated regimes. Under inflation targeting,

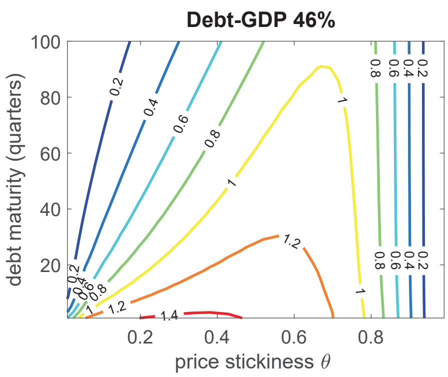

Figure 7: Combinations of price stickiness and debt maturity that generate same welfare loss for two steady-state levels of debt-GDP, 46 percent and 100 percent.

inflation is most stable, but at the cost of high volatility in the output gap. A balanced budget yields the highest inflation volatility, but intermediate volatility in output and losses are the highest, depending on how aggressively fiscal policy returns debt to target in the inflation-targeting regime. Jointly optimal policy strikes a balance between stabilizing inflation and output, to dramatically reduce welfare losses. As debt maturity rises, optimal policy achieves lower output variability at the cost of higher inflation volatility.

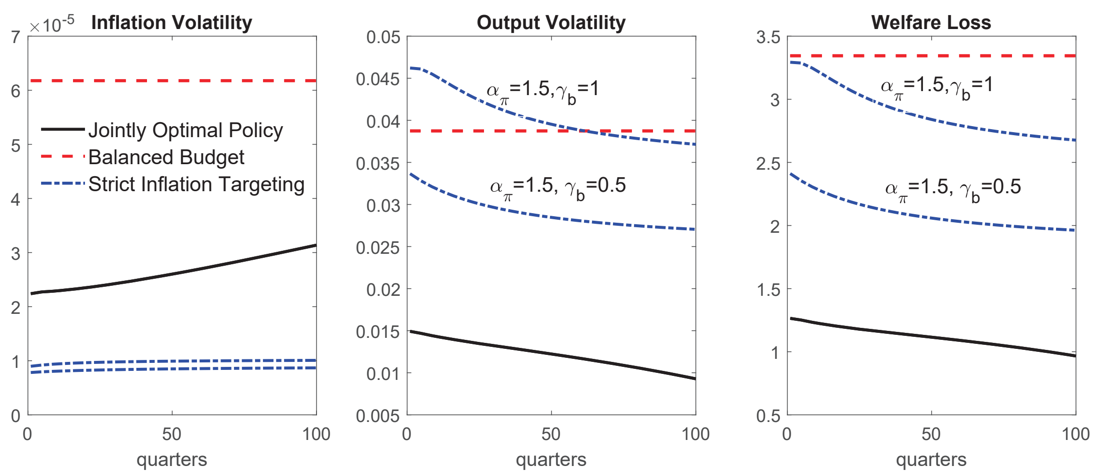

Figure 8: Volatilities of inflation and the output gap and welfare losses under jointly optimal, inflation targeting, and balanced budget policies. Debt maturity on x-axis.

9.2. Effective Lower Bound

Bianchi and Melosi (2019) and Bianchi et al. (2020) propose that when monetary policy is approaching the effective lower bound, it may be beneficial for governments to undertake fiscal expansion that is financed by a mix of inflation and tax revenues. We examine this proposal by considering an exogenous increase in lump-sum transfers to households under contrasting policies: jointly optimal and inflation targeting, as in section 9.1.

Figure 9 plots the impulse responses of inflation, output gap, nominal interest rate, and tax rate to an increase in transfers, $\hat{Z}_t$, that is highly persistent—AR coefficient of 0.9—to reflect stimulus packages like those in the last two recessions. In the inflation-targeting regime, higher transfers bring forth higher current and future taxes, which reduce aggregate demand more severely than aggregate supply (dashed lines). Output declines substantially and persistently. Inflation falls only slightly because aggressive inflation targeting reduces the nominal interest rate by a multiple of the decline in inflation. For a central bank that is already flirting with the effective lower bound, it may be impossible to avoid recession.

Jointly optimal policies smooth the output decline with an initial spike in the tax rate, followed by smooth taxes at a far lower level (solid lines). Optimal policy achieves better output outcomes by promising that debt resulting from the increase in fiscal stress will be inflated away over the maturity structure of outstanding debt. This raises expected inflation to reduce the market value of debt.

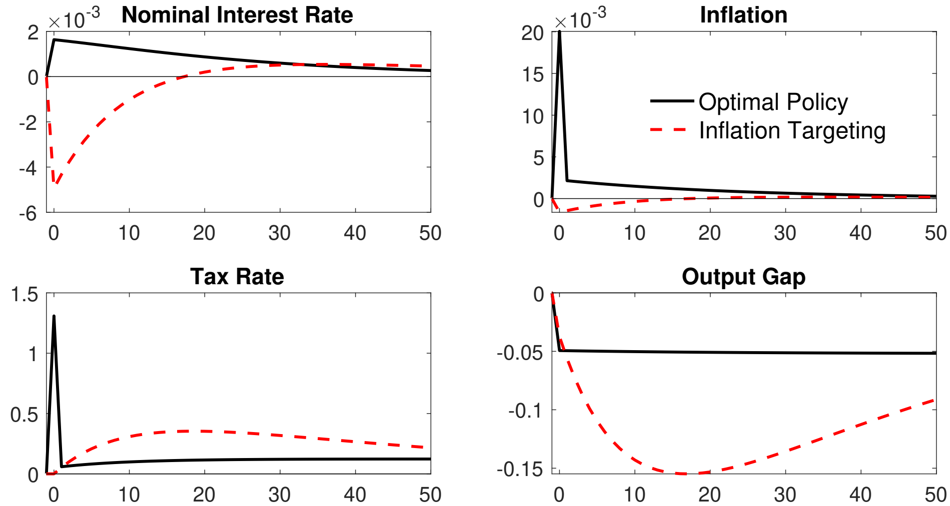

Figure 9: Impulse responses to a one unit increase in fiscal stress, F_t, caused by higher transfers, $\hat{Z}_t$, when average maturity is 5 years. Black solid lines are under jointly optimal policy; red dashed are under the inflation targeting regime with $\alpha_\pi = 1.5$ and $\gamma_b = 0.5$ in the rules the beginning of section 9 describes.

10. Concluding Remarks

The paper establishes that both the maturity structure of government debt and the level of debt matter for optimal monetary and fiscal choices with commitment. Quantitative counterfactual exercises find that on the margin jointly optimal policy would have financed 14 percent of U.S. fiscal needs with current and expected inflation after World War II and 7 percent with inflation after the financial crisis. With higher levels of debt or longer maturity debt like those in tables advanced economies experience today, those financing shares would have been close to 50 percent.

Of course, policy cannot fully commit, as this paper's analysis assumes. A key difference between our commitment results and time-consistent policies is summarized by Leeper et al. (2021, p. 601): "The temptation to use inflation surprises to stabilize debt grows with the level of debt and shrinks with the average maturity of that debt. As a result the equilibrium inflationary bias problem can be significantly lower with longer maturity debt." With commitment, inflation's role in fiscal finance increases with debt maturity. But central banks are adopting frameworks—like inflation targeting—and tools—like forward guidance—that aim to enhance the ability of monetary policy to anchor expectations through commitment. Simultaneously, the fiscal rules that countries are increasingly embracing contribute to enhanced fiscal commitment [International Monetary Fund (2009, 2017)]. This paper argues that the maturity structure of publicly held debt deserves a prominent place in those policy designs.

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