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Optimal Time-Consistent Monetary, Fiscal and Debt Maturity Policy☆

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ABSTRACT

The textbook optimal policy response to an increase in government debt is simple—monetary policy should actively target inflation, and fiscal policy should smooth taxes while ensuring debt sustainability. Such policy prescriptions presuppose an ability to commit. Without that ability, the temptation to use inflation surprises to offset monopoly and tax distortions, as well as to reduce the real value of government debt, creates a state-dependent inflationary bias problem. High debt levels and short-term debt exacerbate the inflation bias. But this produces a debt stabilization bias because the policy maker wishes to deviate from the tax smoothing policies typically pursued under commitment, by returning government debt to steady-state. As a result, the response to shocks in New Keynesian models can be radically different, particularly when government debt levels are high and maturity short.

1. Introduction

Conventional monetary-fiscal policy analysis assigns monetary policy the task of controlling demand and inflation and fiscal policy the job of ensuring fiscal sustainability. Optimal policy analyses support this policy assignment. In sticky price New Keynesian models with one-period government debt, Schmitt-Grohe and Uribe (2004b) show that even a mild degree of price stickiness implies negligible use of inflation surprises to stabilize debt and near random walk behavior in government debt and tax rates when policy makers can commit to time-inconsistent monetary and fiscal policies, in response to shocks. In other words, monetary policy should be used to stabilize inflation, not debt, while a tax smoothing fiscal policy ensures fiscal sustainability. Although Sims (2013) questions the robustness of this result when government can issue long-term

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nominal bonds since this implies variations in bond prices can be used as a device to stabilize debt, Faraglia et al. (2013), Leeper and Leith (2016), Leeper and Zhou (2013) and Sheedy (2014) find that, as part of a Ramsey problem, the consensus policy assignment remains largely optimal - the use of inflation to stabilize debt is negligible.

Relaxing the assumption that the policy maker can commit in a model with single-period debt, Niemann et al. (2013) find that the desire to inflate away the debt burden leads to large and persistent movements in inflation which are absent under commitment. The current paper also assumes time-consistent policy making and assesses the importance of both debt maturity and the level of debt for the resulting equilibrium. Three key findings emerge:

1. The temptation to use inflation surprises to stabilize debt grows with the level of debt and shrinks with the average maturity of that debt. As a result the equilibrium inflationary bias problem can be significantly lower with longer maturity debt.
2. The response to shocks is radically different under discretion vs. commitment and depends crucially on the level and maturity of government debt. Under commitment (regardless of the level and maturity of debt) the policy maker sustains debt at a new steady-state level after effectively eliminating the inflationary consequences of any shock. Under discretion, the policy response is radically different - the debt-dependent inflationary bias leads the policy maker to more than offset the fiscal consequences of the shock to avoid exacerbating these biases. This perverse policy response is heightened for higher debt levels or shorter maturity.
3. Allowing the policy maker to choose the relative proportions of short- versus long-term debt as part of the time-consistent policy problem provides the current policy maker with a means to influence the pace at which a given stock of debt is reduced in the future. When the inflationary bias problem bites less (when prices are more flexible and markups lower) the policy maker will seek to issue less short-term debt, thereby increasing average debt maturity. Since the debt stabilization bias rises in debt levels, but falls in maturity, this helps ensure future policy makers stabilize debt more slowly and at a lower inflationary cost. Conversely, when the inflation bias problem is greater, issuing more short-term debt helps ensure future policy makers stabilize debt more rapidly.

Aside from the key papers cited above which analyze the policy problem in the context of New Keynesian models, monetary frictions have been used to generate a cost for inflation and generate trade-offs between the use of monetary and fiscal policy. For example, Schmitt-Grohe and Uribe (2004a) study Ramsey policy in a flexible-price model with a cash-in-advance constraint, while (Martin, 2009) studies the time-consistency problems that arise from the interaction between debt and monetary policy, since inflation surprises reduce the real value of nominal liabilities. Martin (2011, 2013, 2014) examine time-consistent policies in variants of the monetary search model of Lagos and Wright (2005). Niemann et al. (2013) combines both a cash-in-advance constraint and sticky prices in the context of time-consistent policy with single-period debt - the monetary friction helps ensure the model can sustain a positive debt-to-GDP ratio in steady-state. Monetary frictions are considered in Online Appendix 7, but most of the analysis abstracts from such frictions and emphasizes nominal price stickiness as the conventional approach to generating sizable real effects from monetary policy.

The paper proceeds as follows. The benchmark model is described in Section 2 and the optimal time-consistent policy problem is contrasted with Ramsey policy in Section 3. Section 4 describes the solution method and Section 5 presents the numerical results. Section 6 concludes.

2. The model

The model is a standard New Keynesian model, but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or long-term borrowing.[1]

2.1. Households

The utility function of the representative household takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{N_t{}^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

Households appreciate private consumption, C_t, as well as the provision of public goods, G_t, and dislike supplying labor, N_t. Private consumption is made up of a basket of goods defined by,

$$C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{2}$$

where j denotes the good's variety and $\epsilon_t > 1$ is the elasticity of substitution between varieties. This is assumed to be time-varying, following the AR(1) process,

$$\ln(\epsilon_t) = (1-\rho_\epsilon)\ln(\overline{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \varepsilon_t \sim N(0,1) \tag{3}$$

[1] Most countries issue long-term nominal debt such that even modest changes in inflation and interest rates can have substantial impact on the market value of debt - see Hall and Sargent (2011) and Sims (2013) for the empirical findings on the contribution of this kind of fiscal financing to the decline in the U.S. debt-to-GDP ratio from 1945 to 1974.

as a device for introducing mark-up shocks.

The households' optimal allocation of consumption across individual goods implies their demand for good j,

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} C_t$$

where $P_t(j)$ is the price of good j and the aggregate price level is defined as, $P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj\right)^{\frac{1}{1-\epsilon_t}}$.

The budget constraint at time t is given by

$$P_t^M B_t^M \leq \Xi_t + (1+\rho P_t^M) B_{t-1}^M + W_t N_t (1-\tau_t) - P_t C_t + Tr_t \tag{4}$$

where $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$, Ξ_t is the representative household's share of profits in the imperfectly competitive firms producing these goods, W_t are wages, and τ_t is an wage income tax rate. There is also an exogenous fiscal transfer to the household, $Tr_t = P_t tr$, which is introduced to ensure the model reflects the data in terms of the breakdown of fiscal expenditures into public consumption and transfers.[2] In period t households buy government bonds, B_t^M, at price P_t^M, which, following Woodford (2001), are actually a portfolio of many bonds which pay a declining coupon of ρ^j dollars $j+1$ periods after they were issued, where $0 < \rho \leq \beta^{-1}$. A measure of the duration of the bond is given by $(1-\beta\rho)^{-1}$, which allows calibration of ρ to capture the observed maturity structure of government debt.[3] Households bring nominal wealth of $(1+\rho P_t^M) B_{t-1}^M$ into period t.

Households maximize utility subject to the budget constraint (4) to obtain the optimal allocation of consumption across time and price the declining payoff consols,

$$\beta E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(1+\rho P_{t+1}^M\right) \right\} = P_t^M \tag{5}$$

It is convenient to define the stochastic discount factor (for nominal payoffs) for later use, $Q_{t,t+1} \equiv \beta \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right)$where $E_t Q_{t,t+1} = R_t^{-1}$ is the inverse the short-term interest rate which is the policy instrument of the monetary authority.

The second first order condition (FOC) relates to their labor supply decision and is given by,

$$(1-\tau_t)\left(\frac{W_t}{P_t}\right) = N_t^{\varphi} C_t^{\sigma} \tag{6}$$

That is, the marginal rate of substitution between consumption and leisure equals the after-tax wage rate.

Besides these FOCs, necessary and sufficient conditions for household optimization also require the households' budget constraints to bind with equality. Defining $D_t \equiv (1+\rho P_t^M) B_{t-1}^M$, after imposing the no-arbitrage conditions and the no-Ponzi-game condition, the transversality condition can be written as,

$$\lim_{T\to\infty} E_t[Q_{t,T} D_T] = 0 \tag{7}$$

2.2. Government

Aggregate public consumption takes the same form as private consumption,[4]

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{8}$$

such that government demand for individual goods is given by,

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} G_t$$

Government expenditures, consisting of transfers, Tr_t, and consumption, G_t, are financed by levying labor income taxes at the rate τ_t, and by issuing long-term bonds B_t^M. The government's sequential budget constraint is then given, in real terms, by

$$P_t^M b_t = (1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t} - w_t N_t \tau_t + G_t + tr \tag{9}$$

[2] It is important to note that real transfers are an exogenously given constant and are not considered to be a policy instrument. Allowing transfers to be chosen optimally would enable the policy maker to levy a lump-sum tax in order to finance a negative distortionary labor income tax and offset the distortion arising from monopolistic competition. This is a common, but unrealistic, assumption in linear-quadratic analyses of optimal fiscal and monetary policy in New Keynesian models.

[3] In the special case where $\rho = 0$, the bonds reduce to the familiar single period bonds typically studied in the literature.

[4] An alternative modeling approach would be to introduce an 'aggregator' firm which converts the individual goods to a final output which is purchased by households and the government. The model implies, equivalently, that households and the government perform this aggregation themselves.

where $w_t = W_t/P_t$ is the real wage and $\Pi_t \equiv P_t/P_{t-1}$ the gross rate of inflation. Transfers $tr = Tr_t/P_t$ are fixed at a data-consistent average. It is important to note that the state variable, $b_t \equiv B_t^M/P_t$, which deflates the number of nominal bonds by the price level does not capture the real value of government debt. That is given by, $P_t^M b_t$. Instead, introducing the variable b_t enables the policy problem to be written solely in terms of this state variable without the need to account for B_t^M and P_t.[5]

2.3. Firms

Firm j faces three constraints, firstly a linear production function,

$$Y_t(j) = N_t(j) \tag{10}$$

where the real marginal cost of production is defined as $mc_t \equiv W_t/P_t = (1-\tau_t)N_t^{\varphi}C_t^{\sigma}$. Secondly, a demand curve for their product,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t$$

which is the sum of private and public demand, where $Y_t = [\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj]^{\frac{\epsilon_t}{\epsilon_t - 1}}$. Finally, quadratic adjustment costs in changing prices, as in Rotemberg (1982), defined for firm j as,

$$\eta_t(j) \equiv \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{11}$$

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem facing firm j is to maximize the discounted value of nominal profits,

$$\max_{P_t(j)} E_t \sum_{z=0}^{\infty} Q_{t,t+z} \Xi_{t+z}(j)$$

subject to these constraints above, where nominal profits are defined as,

$$\Xi_t(j) \equiv P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

The FOCs imply the following non-linear Phillips curve relationship,

$$\Pi_t(\Pi_t - 1) = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1}(\Pi_{t+1} - 1)\right] + \phi^{-1}((1-\epsilon_t) + \epsilon_t(1-\tau_t)N_t^{\varphi}C_t^{\sigma}) \tag{12}$$

2.4. Market clearing

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + \eta_t(j)$$

such that, in a symmetrical equilibrium,

$$Y_t\left[1 - \frac{\phi}{2}(\Pi_t - 1)^2\right] = C_t + G_t \tag{13}$$

There is also market clearing in the bonds market where the portfolio of long-term bonds held by households evolves according to the government's budget constraint.

That completes the description of the model, which is summarized in Online Appendix 2. Before analyzing the optimal policy problem the competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium)**.** A competitive equilibrium consists of government fiscal policies, $\{G_t, \tau_t, b_t\}_{t=0}^{\infty}$, prices,$\{R_t, w_t, P_t^M, \Pi_t\}_{t=0}^{\infty}$, and private sector allocations, $\{C_t, N_t, Y_t, \Xi_t\}_{t=0}^{\infty}$, satisfying $\forall\{\epsilon_t\}_{t=0}^{\infty}$, (i) the private sector optimization taking government policies and prices as given, that is, the household budget constraint (4), the production function $Y_t = N_t$, and the optimality conditions, (5), (6) and (12); (ii) the market clearing condition (13); (iii) the government's budget constraint (9); and (iv) the transversality condition (7), for a given initial level of government debt b_{-1}.

[5] Retaining B_t^M and P_t as separate states would be impossible to solve using our solution algorithm since both variables would be non-stationary in an equilibrium with positive inflation.

3. Optimal policy under commitment and discretion

This section outlines the policy problems under both commitment and discretion, before contrasting the resultant FOCs to gain insight into the time-consistency problems generated under discretion. In both cases the policy problem amounts to choosing a set of government policies, $\{R_t, G_t, \tau_t, b_t\}_{t=0}^{\infty}$, in order to maximize the utility of the representative household, (1), subject to the constraints implied by the competitive equilibrium defined above. The difference between commitment and discretion lies in whether or not the policy maker is able to commit to future policies

3.1. Commitment Policy

Following Leeper and Leith (2016), Ramsey policy is derived to serve as a benchmark against which to contrast time-consistent policy. The Lagrangian for the policy problem is given by

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} \\ & + \lambda_{1t}\left[Y_t\left(1 - \frac{\phi}{2}(\Pi_t - 1)^2\right) - C_t - G_t \right] \\ & + \lambda_{2t}\left[\begin{array}{l} (1-\epsilon_t) + \epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t(\Pi_t - 1) \\ + \phi\beta C_t^{\sigma} Y_t^{-1} E_t (C_{t+1})^{-\sigma} Y_{t+1}\Pi_{t+1}(\Pi_{t+1}-1) \end{array} \right] \\ & + \lambda_{3t}\left[P_t^M b_t - (1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \right] \\ & + \lambda_{4t}\left[P_t^M - \beta E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \Pi_{t+1}^{-1}\left(1+\rho P_{t+1}^M\right) \right\} \right] \Bigg\} \end{aligned} \tag{14}$$

Here the consumption Euler equation has been used to eliminate the short-run nominal interest rate, R_t, from the remaining constraints. By committing to an entire path of policy instruments, the policy maker is able to influence expectations in order to improve the policy trade-offs they face.

The resultant set of FOCs are given by,

$$\begin{array}{ll} C_t & \begin{array}{l} C_t^{-\sigma} - \lambda_{1t} + \lambda_{2t}\left[\begin{array}{l} \sigma\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} \\ + \sigma\phi\beta C_t^{\sigma-1} Y_t^{-1} E_t (C_{t+1})^{-\sigma} Y_{t+1}\Pi_{t+1}(\Pi_{t+1}-1) \end{array} \right] \\ + \lambda_{3t}\left[\sigma\left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi} C_t^{\sigma-1} \right] \\ - \lambda_{4t}[\sigma\beta E_t\{(C_t)^{\sigma-1}(C_{t+1})^{-\sigma}\Pi_{t+1}^{-1}(1+\rho P_{t+1}^M)\}] \\ - \lambda_{2t-1}[\sigma\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}(C_t)^{-\sigma-1} Y_t \Pi_t(\Pi_t - 1)] \\ + \lambda_{4t-1}\left[\sigma(C_{t-1})^{\sigma}(C_t)^{-\sigma-1}\Pi_t^{-1}(1+\rho P_t^M)\right] = 0 \end{array} \\ Y_t & \begin{array}{l} -Y_t^{\varphi} + \lambda_{1t}\left[1 - \frac{\phi}{2}(\Pi_t - 1)^2\right] + \lambda_{3t}\left[(1+\varphi) Y_t^{\varphi} C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right] \\ + \lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi\beta C_t^{\sigma} Y_t^{-2} E_t (C_{t+1})^{-\sigma} Y_{t+1}\Pi_{t+1}(\Pi_{t+1}-1)\right] \\ + \lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}(C_t)^{-\sigma}\Pi_t(\Pi_t - 1)] = 0 \end{array} \\ \tau_t & \epsilon_t\lambda_{2t} + \lambda_{3t} Y_t = 0 \\ G_t & \chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \\ P_t^M & \begin{array}{l} \lambda_{3t}[b_t - \rho\frac{b_{t-1}}{\Pi_t}] + \lambda_{4t} \\ - \lambda_{4t-1}\left[\rho(C_{t-1})^{\sigma}(C_t)^{-\sigma}\Pi_t^{-1} P_t^M\right] = 0 \end{array} \\ \Pi_t & \begin{array}{l} -\lambda_{1t}[Y_t\phi(\Pi_t - 1)] - \lambda_{2t}[\phi(2\Pi_t - 1)] + \lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}(1+\rho P_t^m)\right] \\ + \lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}(C_t)^{-\sigma} Y_t(2\Pi_t - 1)] \\ + \lambda_{4t-1}\left[(C_{t-1})^{\sigma}(C_t)^{-\sigma}\Pi_t^{-2}(1+\rho P_t^M)\right] = 0 \end{array} \\ b_t & \lambda_{3t} P_t^M - \beta E_t\left[\lambda_{3t+1}\frac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] = 0 \end{array}$$

To obtain a global solution using the algorithm described in Leeper and Leith (2016) define the following state variables, $\tilde{\lambda}_{2t}$ and $\tilde{\lambda}_{4t}$ such that $\lambda_{2t} = \tilde{\lambda}_{2t} C_t^{-\sigma} Y_t$ and $\lambda_{4t} = \tilde{\lambda}_{4t}(C_t)^{-\sigma}$ which allows us to rewrite the FOCs as shown in Online Appendix 4.

The commitment equilibrium is determined by the system given by the FOCs, the constraints in (14), and the exogenous process for the markup shock, (3). The solution to this system is a set of time-invariant equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t, \tilde{\lambda}_{2t-1}, \tilde{\lambda}_{4t-1}\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \tilde{\lambda}_{2t}, \lambda_{3t}, \tilde{\lambda}_{4t}\}$ for all $t \geq 0$. It is the expansion in the set of state variables to include $\tilde{\lambda}_{2t-1}$ and $\tilde{\lambda}_{4t-1}$which captures the commitments made under Ramsey policy.

3.2. Discretionary policy

The policy under discretion seeks to maximize the value function,

$$V(b_{t-1},\epsilon_t)=\max_{C_t,G_t,Y_t,\Pi_t,\tau_t,b_t}\left\{\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g}-\frac{(Y_t)^{1+\varphi}}{1+\varphi}+\widetilde{\beta}E_t[V(b_t,\epsilon_{t+1})]\right\}$$

subject to the resource constraint (13), the New Keynesian Phillips curve (12), and the government's budget constraint, (9) after using labor supply, (6), the production function, (10) and bond price, (5), equations to eliminate N_t, w_t and P_t^M from the constraints. The possibility that the policy maker suffers from a degree of myopia is captured by assuming they may discount the future more heavily than households, $\widetilde{\beta}\le\beta$. A plausible degree of myopia is necessary to ensure the steady-state level of debt under discretion matches the data - this is discussed below.[6]

In conducting this optimization the policy maker is constrained to act in a time-consistent manner. In other words the policy maker cannot make time-inconsistent promises as to how they will behave in the future in order to have a beneficial impact on current policy trade-offs through expectations as they would under Ramsey policy. Instead economic agents anticipate the incentives facing the policy maker in each period and form expectations accordingly. However, the current policy can still influence those expectations by affecting the states the next period's policy maker inherits. To capture this future expectations are replaced by the following state-dependent auxiliary functions,

$$M(b_t,\epsilon_{t+1})\equiv(C_{t+1})^{-\sigma}Y_{t+1}\Pi_{t+1}(\Pi_{t+1}-1) \tag{15}$$

$$L(b_t,\epsilon_{t+1})\equiv(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M) \tag{16}$$

in the NKPC and bond pricing equations, respectively. These functions reflect the fact that, in equilibrium, we can map endogenous variables to the state-space and expectations are formed rationally based on that mapping. The current policy maker, in turn, takes account of this in setting policy. Before deriving the FOCs, it is helpful to define $X_1(b_t,\epsilon_{t+1})\equiv\partial X(b_t,\epsilon_{t+1})/\partial b_t$ for $X=\{L,M\}$ which captures the impact of changing debt on expectations. The Lagrangian for the policy problem can be written as,

$$\begin{aligned}\mathcal{L}=&\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g}-\frac{(Y_t)^{1+\varphi}}{1+\varphi}+\widetilde{\beta}E_t[V(b_t,\epsilon_{t+1})]\\&+\lambda_{1t}\left[Y_t\left(1-\frac{\phi}{2}(\Pi_t-1)^2\right)-C_t-G_t\right]\\&+\lambda_{2t}\left[\begin{array}{c}(1-\epsilon_t)+\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma}-\phi\Pi_t(\Pi_t-1)\\+\phi\beta C_t^{\sigma}Y_t^{-1}E_t[M(b_t,\epsilon_{t+1})]\end{array}\right]\\&+\lambda_{3t}\left[\begin{array}{c}\beta b_tC_t^{\sigma}E_t[L(b_t,\epsilon_{t+1})]-\frac{b_{t-1}}{\Pi_t}(1+\rho\beta C_t^{\sigma}E_t[L(b_t,\epsilon_{t+1})])\\+\left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi}C_t^{\sigma}-G_t-tr\end{array}\right]\end{aligned} \tag{17}$$

where the model equilibrium also requires us to define bond prices, $P_t^M=\beta C_t^{\sigma}E_t[L(b_t,\epsilon_{t+1})]$ since these are embedded in the auxiliary function $L(b_t,\epsilon_{t+1})$. The policy maker optimizes (17) by choosing C_t, G_t, Y_t, Π_t, τ_t, b_t and the multipliers, λ_{1t}, λ_{2t}, λ_{3t}. It should be noted that even though the policy maker optimizes with respect to all endogenous variables, they are not acting as a social planner. Instead, they are choosing standard policy instruments in order to influence the decentralized equilibrium in a manner which maximizes their objective function subject to the time-consistency constraint. The FOCs for the policy problem are detailed below.

$$\begin{array}{ll}C_t & \begin{array}{l}C_t^{-\sigma}-\lambda_{1t}+\lambda_{2t}\left[\sigma\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1}+\sigma\phi\beta C_t^{\sigma-1}Y_t^{-1}E_t[M(b_t,\epsilon_{t+1})]\right]\\+\lambda_{3t}\left[\sigma\beta b_tC_t^{\sigma-1}E_t[L(b_t,\epsilon_{t+1})]-\rho\sigma\beta\frac{b_{t-1}}{\Pi_t}C_t^{\sigma-1}E_t[L(b_t,\epsilon_{t+1})]+\sigma\left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi}C_t^{\sigma-1}\right]=0\end{array}\\Y_t & \begin{array}{l}-Y_t^{\varphi}+\lambda_{1t}\left[1-\frac{\phi}{2}(\Pi_t-1)^2\right]+\lambda_{3t}\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right]\\+\lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma}-\phi\beta C_t^{\sigma}Y_t^{-2}E_t[M(b_t,\epsilon_{t+1})]\right]=0\end{array}\\\tau_t & \epsilon_t\lambda_{2t}+\lambda_{3t}Y_t=0\\G_t & \chi G_t^{-\sigma_g}-\lambda_{1t}-\lambda_{3t}=0\\\Pi_t & -\lambda_{1t}[Y_t\phi(\Pi_t-1)]-\lambda_{2t}[\phi(2\Pi_t-1)]+\lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}(1+\rho\beta C_t^{\sigma}E_t[L(b_t,\epsilon_{t+1})])\right]=0\\b_t & \begin{array}{l}-\widetilde{\beta}E_t\left[\lambda_{3t+1}\frac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right]+\lambda_{2t}\left[\phi\beta C_t^{\sigma}Y_t^{-1}E_t[M_1(b_t,\epsilon_{t+1})]\right]\\+\beta\lambda_{3t}\left[C_t^{\sigma}E_t[L(b_t,\epsilon_{t+1})]+b_tC_t^{\sigma}E_t[L_1(b_t,\epsilon_{t+1})]-\rho\frac{b_{t-1}}{\Pi_t}C_t^{\sigma}E_t[L_1(b_t,\epsilon_{t+1})]\right]=0\end{array}\end{array}$$

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[6] An alternative device for delivering positive steady-state debt levels, which has been used in the literature (see for example Schmitt-Grohe and Uribe, 2004b and Niemann et al., 2013), is to introduce a monetary friction. We consider this approach in Online Appendix 7.

The discretionary equilibrium is determined by the system given by the FOCs, the constraints in (17), the auxiliary equations, (15) and (16), bond prices, $P_t^M = \beta C_t^\sigma E_t[L(b_t, \epsilon_{t+1})]$, and finally the exogenous process for the markup shock, (3). The solution to this system is a set of time-invariant Markov-perfect equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ for all $t \geq 0$.

Further insight into the trade-offs facing the policy maker can be generated by considering specific FOCs, which can also be contrasted with those implied by commitment outlined above. The FOC for taxation,

$$\epsilon_t \lambda_{2t} + \lambda_{3t} Y_t = 0$$

identical under both commitment and discretion, reveals a key feature of the underlying policy problem. In the absence of a need to satisfy the budget constraint through distortionary taxation, $\lambda_{3t} = 0$, the tax instrument would be used to eliminate the costs associated with the output-inflation trade-off implicit in the NKPC, $\lambda_{2t} = 0$. In other words, if it were not for the need to raise tax revenues to satisfy the government's budget constraint, taxes could be adjusted to eliminate any undesired movements in inflation arising from mark-up shocks.

Similarly, the FOC for inflation highlights the nature of the inflationary bias contained in the model,

$$\begin{aligned} 0 = & -\lambda_{1t}[Y_t \phi(\Pi_t - 1)] - \lambda_{2t}[\phi(2\Pi_t - 1)] \\ & + \lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1 + \rho P_t^M\right)\right] \end{aligned} \tag{18}$$

The first two terms of the FOC capture the standard inflationary bias problem. The first term measures the costs of raising inflation, and the second term the output benefits of doing so (given inflationary expectations) which are evaluated positively when the economy operates at a suboptimally low level due to tax and monopolistic competition distortions. However, in the presence of debt the third term in the FOC for inflation captures an additional reason for wanting to raise inflation relative to expectations - the erosion of the real value of debt. Economic agents will anticipate that higher debt increases the government's desire to introduce inflation surprises, implying that inflationary expectations in the NKPC are increasing in the level of government debt, $E_t[M_1(b_t, \epsilon_{t+1})] > 0$ until inflation is sufficiently high to eliminate policy surprises (in the absence of further shocks). The state dependence of the inflationary bias will be key in driving the policy maker's desire to reduce debt relative to what would be observed under a time-inconsistent Ramsey policy - a tendency we label the "debt stabilization bias".

The three terms in (18) are common to the FOC for inflation under both discretion and commitment, but where the latter contains the following additional terms,

$$\begin{aligned} & +\lambda_{2t-1}[\phi C_{t-1}^\sigma Y_{t-1}^{-1} (C_t)^{-\sigma} Y_t (2\Pi_t - 1)] \\ & +\lambda_{4t-1}\left[(C_{t-1})^\sigma (C_t)^{-\sigma} \Pi_t^{-2} (1 + \rho P_t^M)\right] \end{aligned}$$

The first captures the extent of the policy maker's commitment not to raise inflation in an attempt to raise output; this commitment reduces inflationary expectations. The second their commitment not to use inflation to reduce bond prices. The numerical analysis below suggests that the state-dependent inflationary bias is significant, but that, if able, the policy maker would largely commit to not using inflation as a device to stabilize debt.

The remaining key FOC is for government debt which highlights the "debt stabilization bias". This bias can be understood by considering the FOC for debt, which can be simplified as,

$$\begin{aligned} & \underbrace{P_t^M \lambda_{3t} - \widetilde{\beta} E_t\left[\frac{\lambda_{3t+1}}{\Pi_{t+1}}(1 + \rho P_{t+1}^M)\right]}_{\text{tax smoothing}} \\ & \underbrace{-\lambda_{3t} C_t^\sigma \left(\phi \epsilon^{-1} \beta E_t[M_1(b_t, \epsilon_{t+1})] - \left[(b_t - \rho \frac{b_{t-1}}{\Pi_t}) E_t L_1(b_t, \epsilon_{t+1})\right]\right)}_{\text{debt stabilization bias}} = 0 \end{aligned} \tag{19}$$

Eq. (19) describes the policy maker's optimal debt policy which can be decomposed into two elements. The first line gives the optimal trade-off between current and future distortions associated with the need to satisfy the government's intertemporal budget constraint - tax smoothing. These terms are present under both commitment and discretion. The second line, captures wedges which are introduced when the policy maker is unable to commit, defining the debt stabilization bias.[7] It is helpful to discuss the implications of 'tax smoothing', before assessing how that policy is impacted by the 'debt stabilization bias' generated by an inability to commit.

The tax-smoothing argument in Barro (1979), requiring that the marginal costs of taxation are smoothed over time, is reflected in the relationship between λ_{3t} and λ_{3t+1} in the first line of (19). Initially assume the policy maker is not myopic, so $\widetilde{\beta} = \beta$. In this case, when the return (adjusted for any covariance with the future costs of satisfying the government's

[7] The remaining FOCs determine the policy mix employed to achieve the debt dynamics implied by the debt stabilization bias, (19) which, in turn, is driven by the state-dependent inflationary bias problem, (18).

intertemporal budget constraint, λ_{3t+1}) on holding the government bonds is equal to the household's rate of time preference, the distortions associated with satisfying the budget constraint are constant in steady-state and steady-state debt will follow a random walk. Effectively, under tax smoothing, the policy maker trades-off the short-run costs of reducing the stock of debt against the discounted value of the long-term benefits of lower debt. When debt service costs are consistent with the household/government's rate of time preference, as they are in steady-state, these will be exactly balanced at a debt level which depends upon the history of the shocks hitting the economy.

Reintroducing myopia, such that $\widetilde{\beta} < \beta$, implies that when real interest rates differ from the policy maker's rate of time preference, then the policy maker will choose to tilt these distortions backwards (forwards) in time depending on whether debt service costs are below (above) the policy maker's rate of time preference. For example, when the real rate of return on debt, $r_t^b \equiv E_t\left[\frac{1}{\Pi_{t+1}}\frac{(1+\rho P_{t+1}^M)}{P_t^M}\right] = \beta^{-1}$, this implies $E_t\left[\frac{\lambda_{3t+1}}{\lambda_{3t}}\frac{1}{\Pi_{t+1}}\frac{(1+\rho P_{t+1}^M)}{P_t^M}\right] = \widetilde{\beta}^{-1} > \beta^{-1}$ such that λ_{3t} is rising over time. The myopic policy maker would allow debt to rise.

However under discretion, (19) is a generalized Euler equation, which, in the second line, includes partial derivatives of policy functions with respect to debt due to the time-consistency requirement. In general the form of these auxiliary functions is unknown, which is why the policy problem needs to be solved numerically. However, that numerical solution robustly gives clear signs for these derivatives, $M_1(b_t, \epsilon_{t+1}) > 0$ and $L_1(b_t, \epsilon_{t+1}) < 0$ which have an intuitive interpretation.

The first term on the second line of (19) reflects the fact that inflation expectations rise with debt levels (through the inflation biases discussed above - see the FOC for inflation), $M_1(b_t, \epsilon_{t+1}) > 0$, and since this is costly in the presence of nominal inertia, there is a desire to deviate from tax smoothing, in order to reduce debt and the associated increase in inflation. This is the first reason for wanting to reduce debt relative to the level that would be supported by a benevolent Ramsey planner.

The second term in square brackets in the second line captures the impact of higher debt on bond prices. Since higher debt raises inflation, which in turn reduces bond prices, $L_1(b_t, \epsilon_{t+1}) < 0$, this term also serves to encourage a reduction in debt levels, when debt is relatively short-term. Why? High, but falling debt levels imply an upward trend in bond prices which makes it cheaper to issue new debt, but more costly to buy-back the existing debt stock. As debt maturity is increased, the latter effect rises relative to the former, and hence the desire to reduce debt levels is reduced, ceteris paribus. This trade-off between tax-smoothing and time-consistency determines the equilibrium level of debt and inflation, where inflation is expected to be closer to zero as debt maturity rises, for a given level of debt.[8]

4. Solution method and calibration

For the model described in the previous section, the equilibrium policy functions cannot be computed in closed form and local approximation methods are not applicable, as the model's steady state around which local dynamics should be approximated is endogenously determined as part of the model solution and thus is a priori unknown. This necessitates the use of global solution methods. Specifically, the Chebyshev collocation method with time iteration. The detailed algorithm is presented in Online Appendix 5. In general, optimal discretionary policy problems can be characterized as a dynamic game between the private sector and successive governments. Multiplicity of equilibria is a common problem in dynamic games of this kind. Since the solution algorithm uses polynomial approximations, it is, in effect, searching only for continuous Markov-perfect equilibria where agents condition their strategies on payoff-relevant state variables, see Judd (2004) for a discussion.

Before solving the model numerically, the benchmark values of structural parameters must be specified. The calibration of parameters is summarized in Table A.1. We set $\beta = (1/1.02)^{1/4} = 0.995$, which implies a 2% annual real interest rate. The intertemporal elasticity of substitution is set to one half ($\sigma = \sigma^g = 2$) which is in the middle of standard estimates.[9] The Frisch labor supply elasticity is set to $\varphi^{-1} = 1/3$. The steady-state elasticity of substitution between intermediate goods is chosen as $\overline{\epsilon} = 14.33$, which implies a monopolistic markup of approximately 7.5%, similar to Siu (2004), and in the middle of conventional estimates.

The fiscal variables are calibrated to ensure the benchmark model mimics the key ratios in U.S. data over the period 1954-2008 as discussed in the Online Appendix 1 and reported in the first column of Table A.2. Parameter $\chi = 0.0076$ ensures government consumption is 7.8% of GDP, transfers are set to be 9% and the myopia of the policy maker is set to $\widetilde{\beta} = 0.982$ (an effective time horizon of just under 20 years) which supports an annualized steady-state debt-to-GDP ratio of 31%. The coupon decay parameter, $\rho = 0.95$, corresponds to around 5 years of debt maturity, consistent with U.S. data. The implied ratio of tax revenues to GDP in steady-state is slightly higher than the data average of 17.5% reflecting the fact that actual policy has often run a deficit in recent decades.

The price adjustment cost parameter, $\phi = 50$, implies, given the equivalence between the linearized NKPCs under Rotemberg and Calvo pricing (see Leith and Liu, 2016), that on average firms re-optimize prices every six months - in line with

[8] For completeness the opposite cases $M_1(b_t, \epsilon_{t+1}) < 0$ and $L_1(b_t, \epsilon_{t+1}) > 0$, should be considered. However this would imply that higher debt reduced inflationary expectations and raised bond prices. This in turn would encourage the policy maker to deviate from the policy of tax smoothing by raising rather than lowering debt. This non-intuitive case is not something ever observed in the numerical analysis.

[9] In the robustness exercises conducted in Online Appendix 8 the elasticity for public spending is lowered in line with the evidence in Debortoli and Nunes (2013). However, this does not affect the key results.

empirical evidence. Finally, the cost-push shock process is parameterized as $\rho_\epsilon = 0.939$ and $\sigma_\epsilon = 0.052$ in line with estimates in Chen et al. (2017) and Smets and Wouters (2003).

With this benchmark parameterization, the model solution generates a maximum Euler equation error over the full range of the grid is of the order of 10^{-6}. We plot these errors in Online Appendix 6. As suggested by Judd (1998), this order of accuracy is reasonable.[10]

5. Numerical results

This section explores the properties of the equilibrium under optimal time-consistent policy. Section 5.1 considers the steady-state under a series of alternative parameterizations. Section 5.2 contrasts the policy response to shocks under commitment and discretion, and how debt maturity affects those differences. Section 5.3 does the same for the level of debt and the debt-maturity decision is endogenized in Section 5.4.

5.1. Steady state

Table A.2 summarizes the steady state values for a variety of parameterizations, contrasting them with the data averages contained in column 1. The analysis begins with the benchmark calibration after temporarily removing policy maker myopia such that $\widetilde{\beta} = \beta$ - column 3 of Table A.2. The key trade-offs underpinning this steady-state equilibrium can been seen by considering the (deterministic) steady-state value of FOC for debt, Eq. (19),

$$b(1 - \rho\frac{1}{\Pi})L_1(b, \overline{\epsilon}) = \phi\overline{\epsilon}^{-1}\beta M_1(b, \overline{\epsilon}) \tag{20}$$

As noted above, the numerical solution of the policy problem implies $L_1(b, \overline{\epsilon}) < 0$ and $M_1(b, \overline{\epsilon}) > 0$. Assuming $\rho < \Pi$, this equation can only hold with a negative debt stock.[11] This is indeed what happens with $\frac{bP^M}{4Y} = -153\%$ and a steady-state inflation rate of -1.1%. The reason for the deflation can be seen from the FOC for inflation, Eq. (18),

$$\lambda_1[Y\phi(\Pi - 1)] = \lambda_3\left[Y\phi\epsilon^{-1}(2\Pi - 1) + \frac{b}{\Pi^2}\left(1 + \rho\frac{\beta}{\Pi - \rho\beta}\right)\right] \tag{21}$$

which equates the resource costs of a marginal increase in inflation with the marginal benefits in terms of higher output and reduced debt of an inflation surprise. For a positive value of debt and suboptimally low level of equilibrium output, the inflation bias will be positive. However, as debt turns negative the marginal benefits of inflation surprises fall as this reduces the value of the government's assets. If debt turns sufficiently negative, the equilibrium supports a steady-state deflation which ensures the policy maker is not tempted to introduce any further surprise deflation to increase the value of the assets she has accumulated. As a result the accumulated assets fall short of the war chest level needed to support the first best allocation.[12]

Introducing policy maker myopia can overturn this result - see the second column of Table A.2, labeled "benchmark". The intuition is as follows - within line one of (19) the myopic policy maker weighs the costs of debt reduction more than the long-term benefits, thereby tilting the tax smoothing element of optimal debt policy towards rising debt levels. This is then balanced against the existing debt stabilization bias to deliver a higher equilibrium level of debt, cet. par.. By introducing myopia, the benchmark has been calibrated to replicate a positive debt-to-GDP ratio of 31% and government consumption to output of 7.8%. The steady-state rate of inflation this implies is 3%. The key equation defining this steady-state is the FOC for debt given by

$$b(1 - \rho\frac{1}{\Pi})L_1(b, \overline{\epsilon}) = \phi\overline{\epsilon}^{-1}\beta M_1(b, \overline{\epsilon}) - C^{-\sigma}P^M(1 - \frac{\widetilde{\beta}}{\beta}) \tag{22}$$

where the myopia can turn the RHS of this condition negative, thereby supporting a positive steady-state debt-to-GDP ratio. It is notable that this change does little to affect the other key fiscal ratios of government consumption and taxation relative to GDP. Column 4 increases policy maker myopia further to $\widetilde{\beta} = 0.975$, which is equivalent to reducing the policy maker's time horizon from 20 to 12 years. This more than doubles the steady-state debt-to-GDP ratio to 75.6% and inflation rises to 4.5%.

Increasing the flexibility of prices means both that the costs of inflation are lower and that monetary policy has smaller effects on the real economy - see the first two terms in the FOC for inflation, (18), respectively. This has the effect of making the inflationary bias problem less costly which reduces the debt-stabilization bias. As a result the government is able to sustain a higher debt-to-GDP ratio which rises by 5.5%, as they are less driven to reduce the state-dependent inflationary bias problem. This leads to a larger steady-state rate of inflation of 3.8%, but it should be remembered that inflation is now less costly, so that moderates the inflationary bias problem.

[10] All other model variants considered are equally well approximated - these results are available upon request.

[11] No parameter permutations have been found which imply $\rho > \Pi$ such that the model without myopia can sustain a positive steady-state debt stock. Intuitively, unless debt stocks are negative, the economy remains sufficiently distorted that the inflationary bias problem ensures $\Pi > \rho$.

[12] The war chest asset stock would be 4,636% of GDP - see Online Appendix 3.

Finally, reducing the mark-up (from 7.5% to 5% in the final column of Table A.2) is important since it implies the inflationary bias problem is lower for a given level of debt. (The gains to a surprise inflation are lower, when the economy is less distorted - see the impact of a higher value of ϵ in the second term of the FOC for inflation, (18)). As a result the desire to influence the state-dependent inflationary bias problem by reducing debt is less - the debt stabilization bias has been reduced. This substantially increases the steady-state debt-to-GDP ratio to almost 90% and the steady-state rate of inflation to 3.7%.

Table A.3, considers the impact of changes in the maturity structure of debt. Column 1 adopts the common assumption that debt is only of a single period's duration (one quarter in the context of the model parameterization). In this case the steady-state debt-to-GDP ratio turns negative, −11% and inflation is 3.5%. Increasing debt maturity to 30 years leads to a significant increase in the debt-to-GDP ratio to over 102% of GDP and inflation to over 5%. This reflects the discussion above - longer maturity debt reduces the debt stabilization bias allowing the government to sustain a higher steady-state debt-to-GDP ratio.

In summary, myopia, monopolistic competition distortions and debt maturity are the key drivers of the equilibrium rate of inflation and debt-to-GDP ratio, while other endogenously determined steady-state fiscal ratios are largely unaffected by these changes. This highlights the importance of the state-dependent inflationary bias and the associated debt stabilization bias in jointly determining the equilibrium outcomes for inflation and debt.

5.2. Responding to shocks - debt maturity

This subsection contrasts how the policy maker responds to shocks, under both commitment and discretion with either single-period or long-term debt.[13] Fig. B.1 plots the outcomes for key variables following a rise in the markup $\frac{\varepsilon_t}{\varepsilon_t - 1}$ by 0.5%. Under commitment the policy maker cuts taxes to largely offset the shock, but in the long-run slightly raises taxation in order to sustain (but not reverse) the higher stock of debt that emerges as a result. There is a very limited use of surprise inflation in the short-run to reduce the need to increase taxes in the long-run, but this is small. Debt maturity has a negligible impact, only facilitating a more gradual use of inflation to stabilize debt, but barely noticeably.[14]

The policy outcome under discretion is radically different. The case of single period debt is represented by the red dash-dotted line in Fig. B.1.[15] Although tax cuts could in theory offset the inflationary consequences of the mark-up shock as under commitment, this would exacerbate the increase in debt which drives the inflationary bias problem discussed above. As a result the policy maker *raises* tax rates to ensure that debt falls as a more effective way of mitigating the inflationary consequences of the mark-up shock. Nevertheless the higher tax rates and mark-up shock do increase inflation and monetary policy is tightened to help offset that.[16] The end result is that the response to the mark-up shock is overwhelmingly driven by the desire to reduce debt through tax increases and thereby mitigate the state-dependent inflationary bias problem. Government spending largely moves in line with output such that there is negligible variation in the ratio of G/Y-government consumption is hardly used as an instrument of either macroeconomic or fiscal stabilization.[17]

Although debt maturity has little impact on policy outcomes under commitment it matters a lot under discretion. Longer-term debt significantly reduces the debt-stabilization bias such that the steady-state rate of inflation is significantly lower when debt is of longer maturity (falling by 3%). Moreover, the reduction in the debt-stabilization bias with longer maturity debt also reduces the desire of the policy maker to offset the fiscal repercussions of the mark-up shock, resulting in much more moderate tax increases and monetary policy tightening. As a result debt falls by less than in the case of single period debt.[18]

5.3. Responding to shocks - level of debt

This subsection explores the impact of the level of debt on policy outcomes under commitment and discretion, examining the same mark-up shock as above, but with steady-state levels of debt-to-GDP of 51.5% and 15.8%, respectively. These levels capture the peaks and troughs of the US debt-to-GDP ratio following World War II - see Fig. B.4 in Online Appendix 9.

[13] The time-consistent policy problem with single-period debt has a raised degree of myopia to ensure it shares the same steady-state debt-to-GDP ratio as the benchmark model.

[14] See Leeper and Leith (2016) for a discussion of how surprise inflation can contribute to the stabilization of debt of different maturities.

[15] To ensure comparability, myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the benchmark model.

[16] The gross quarterly real interest rate is defined as $E_t[R_t/\Pi_{t+1}]$ and is plotted in this, and subsequent figures, as a net annualized percentage.

[17] If the intertemporal elasticity of substitution for government consumption is reduced to $\sigma_g = 1$ in line with the evidence summarized in Debortoli and Nunes (2013), the standard deviation of the G/Y ratio rises from 0.8% to 1.4%, which is closer to the data average of 1.9%. However, this does not have a significant impact on any of the experiments conducted in the paper other than to marginally enhance the role played by government consumption. See the Online Appendix 12.

[18] The Online Appendix 8 considers the impact of a government spending shock. This shows a similar pattern of response - under commitment a slight rise in taxation is sufficient to stabilize the debt stock at a permanently higher level, while monetary policy tightens to effectively eliminate inflation. In contrast, discretionary policy acts to reduce the inflationary impact of the shock by reducing debt (and the associated inflationary bias problem) through substantial tax increases, while moderating the tightening of monetary policy.

Online Appendix 9 introduces switches in the degree of policy-maker myopia which enables us to track these movements.[19] However, here the focus is on how debt levels affect the policy response to shocks under commitment and discretion.

Since, under commitment, steady-state debt follows a random walk, when analyzing commitment initial steady-state conditions consistent with these two debt levels are adopted. In contrast, under discretion different steady-state debt-to-GDP ratios can be considered by adopting a high or low myopia regime. Therefore, four scenarios are considered - the impact of a rise in the markup $\frac{\varepsilon_t}{\varepsilon_t - 1}$ of 0.5% under commitment and discretion starting from steady-states with either high or low levels of debt.

The policy response under commitment is essentially the same as before, regardless of the level of debt - see Fig. B.2. In the short to medium term, taxes fall to offset the mark-up shock, but eventually rise to sustain the higher stock of debt that emerges as a result. Since there is a negligible tightening of monetary policy, debt dynamics across high and low debt levels are largely unaffected and the rise in the debt-to-GDP ratio as a result of the shock is similar across debt levels.

Under discretion we again find that taxes actually rise following the mark-up shock generating a sustained fall in government debt as a more effective way of moderating inflation than cutting taxes and offsetting the cost-push shock directly. Here, the fact that high debt levels worsen the inflationary bias problem means that the desire to reduce debt is more pronounced the higher the level of debt. Therefore, higher debt levels and, as shown above, shorter debt maturity are more likely to give rise to perverse tax and debt policy responses which move in the opposite direction to those observed under commitment.

5.4. Debt management and the debt stabilization bias

Up until this point, the level of debt maturity has been held fixed by parameterizing ρ. This subsection allows for the policy maker to have some control over the maturity structure as part of the time-consistent optimal policy problem, by allowing them to issue a mixture of short and long-maturity debt, possibly of opposite signs (i.e. one can be held as an asset and the other a liability). Before adding this extra element to the benchmark model, in order to identify the trade-offs facing the policy maker in such an environment a simple three period perfect foresight model is analyzed where the policy maker chooses the mix of one and two period bonds issued in the first period.

5.4.1. 3 Period model

Online Appendix 10 derives the model and policy problem in full. In period $t = 0$ the government can issue a mixture of one and two period bonds following the budget constraint,

$$Q_{0,1}b_{0,1} + Q_{0,2}b_{0,2} = -\tau_0 + \zeta_0 + b_{-1,0}\nu_0 + Q_{0,1}b_{-1,1}\nu_0 \tag{23}$$

where $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation, $Q_{t,t+j}$ is the price of zero coupon debt in period t, which matures in period $t + j$ and the state variables are defined as, $b_{j,k} \equiv B_{j,k}/P_j$ reflecting the quantity of zero coupon nominal bonds issued in period j which mature in period k, deflated by the price level in period j. The perfect foresight equilibrium path follows an initial perturbation generated by transfers in period 0 being $\zeta_0 > 0$, relative to their value of zero in all other periods. Taxes are τ_0 and there is no government consumption.

The economy is an endowment economy with no government consumption such that private consumption always equals its endowment and bond pricing equations are given by, $Q_{t,t+1} = \beta\nu_{t+1}$ and $Q_{t,t+2} = \beta^2\nu_{t+1}\nu_{t+2}$, for one and two-period bonds respectively. The government budget constraints in periods $t = 1$ and 2, are given, respectively, by,

$$Q_{1,2}b_{1,2} = -\tau_1 + \nu_1 b_{0,1} + Q_{1,2}\nu_1 b_{0,2} \tag{24}$$

and,

$$\tau_2 = \nu_2 b_{1,2} \tag{25}$$

Therefore in the second period, $t = 1$, the government can only issue one period bonds and in the final period the government must repay all outstanding debt. Combining the flow budget constraints and bond pricing equations yields the government's intertemporal budget constraint,

$$\zeta_0 + b_{-1,0}\nu_0 + b_{-1,1}\nu_0\nu_1 = \tau_0 + \beta\tau_1 + \beta^2\tau_2$$

which shows that inflation in periods $t = 0$ and 1 acts upon the value of one- and two-period bonds inherited from period $t = -1$, but that the transfers shock ζ_0 needs to be fully funded by taxation.

Following Leeper and Leith (2016) it is assumed that inflation and taxation are costly, such that social welfare is given by,

$$-E_0 \sum_{t=0}^{2} \beta^t \left(\tau_t^2 + \theta(\nu_t - 1)^2\right) \tag{26}$$

[19] Conventional economic shocks cannot mimic the data in this respect. The standard deviation of the annualized debt-to-GDP ratio is only 0.7% under the benchmark calibration despite the equivalent volatility in the data being 9%. Even allowing for temporarily unstable paths for transfers as in Bi et al. (2013) cannot generate data-consistent movements in debt-to-GDP ratios.

The parameter θ captures the relative cost of inflation - a lower value of θ would map to a reduced inflationary bias problem in our benchmark model through more myopia, less price stickiness or lower markups.

Commitment

Online Appendix 10 derives the optimal policy under commitment which implies perfect tax smoothing,

$$\tau_0 = \tau_j \text{ for } j = 1, 2$$

and a pattern of inflation across each period given by,

$$\nu_2 = 1$$

$$-\theta(\nu_1 - 1) = \tau_0 \nu_0 b_{-1,1}$$

and

$$-\theta(\nu_0 - 1) = \tau_0(b_{-1,0} + \beta \nu_1 b_{-1,1})$$

The policy maker commits to zero net inflation in period $t = 2$, and only introduces inflation ($\nu_t < 1$) in periods $t = 0$ and 1 to the extent that she inherits a debt stock which matures in those periods. In the absence of an initial debt stock, $b_{-1,0} = b_{-1,1} = 0$, there would be no net inflation and the policy maker would finance the transfers perturbation solely through taxation, $\zeta_0 = \tau(1 + \beta + \beta^2)$ where τ is the tax-smoothing tax rate applied in each of the three periods. This outcome can be contrasted with that which emerges under the time-consistent policy.

Discretion

The time-consistent policy is solved in Online Appendix 10 by backward induction. In period $t = 2$ the policy maker maximizes period 2 welfare subject to the budget constraint, implying that the optimal policy mix is given by,

$$-\theta \nu_2(\nu_2 - 1) = \tau_2 b_{1,2} \nu_2 \tag{27}$$

This describes the debt-driven inflationary bias - higher levels of debt inherited in period $t = 2$ raise inflation, more so for lower values of θ which imply a reduction in the relative costs of inflation. It should be noted that this inflation does not serve to reduce the real value of debt as it will already have been factored into bond prices when the debt was issued in period 1. Instead the taxes needed to pay off the debt are given by, $\tau_2 = \nu_2 b_{1,2}$.

In period $t = 1$ the policy maker conducts a similar optimization, but treats the period $t = 2$ policy mix, Eq. (27), as an Incentive Compatibility Constraint (ICC) in their optimization. The resultant FOCs are,

$$-\theta \nu_1(\nu_1 - 1) = \tau_1(b_{0,1}\nu_1 + \beta \nu_1 \nu_2 b_{0,2}) \tag{28}$$

$$\tau_2 = \tau_1(2\nu_2 - 1) + \frac{2\tau_1 \tau_2 \nu_1 b_{0,2}}{\theta} \tag{29}$$

The first has the same interpretation as above - the higher the level of debt inherited the greater the inflation and taxation. The second expression guides the period 1 policy maker's optimal rate of debt reduction in order to achieve the desired balance between the current and next period policy mix. If, $b_{0,2} = 0$ and the period $t = 0$ policy maker only issued single-period debt, the period $t = 1$ policy maker would tax more today than the policy maker is required to tax tomorrow, $\tau_1 > \tau_2$ since $\nu_2 < 1$, that is, the debt-stabilization bias causes the policy maker to reduce debt more quickly than tax-smoothing would imply. This rate of correction will be higher as debt levels rise. However, the period $t = 0$ policy maker can influence that behavior by changing the quantity of two period debt they issue. Again, these FOCs will serve as additional ICCs on the period $t = 0$ policy maker.

Now consider the period $t = 0$ policy maker who maximizes the welfare objective (26) subject to the series of budget constraints (23)–(25) and the three ICCs (27)–(29) generated by the policy makers' choices in periods $t = 1$ and $t = 2$. The set of FOCs this implies is detailed in Online Appendix 10. The policy maker will deliver inflation in period $t = 0$ in a similar way to the subsequent policy makers,

$$-\theta \nu_0(\nu_0 - 1) = \tau_0(\nu_0 b_{-1,0} + \beta \nu_0 \nu_1 b_{-1,1}) \tag{30}$$

It is convenient to simplify this and the other FOCs by considering the case where there is no initial stock of debt, $b_{-1,0} = b_{-1,1} = 0$, but the policy maker has to finance a transfers shock, $\zeta_0 > 0$. In this case there would be no inflation in period $t = 0$, $\nu_0 = 1$ and the transfers shock must be entirely financed through taxation, $\zeta_0 = \tau_0 + \beta\tau_1 + \beta^2\tau_2$. In subsequent periods the inflation generated depends upon the quantity of debt remaining, as described by Eqs. (27) and (28), but this does not actually contribute to the financing of that debt as it simply reflects the inflationary bias problem generated by a desire to reduce debt levels through inflation surprises.

The period $t = 0$ policy maker has an additional policy instrument with which to influence the future - the maturity structure of the debt they leave to the future. Online Appendix 10 shows they will choose $b_{0,2}$ to ensure the second ICC generated by the $t = 1$ policy maker's choices (29), does not constrain the time 0 policy problem. Instead, the Online Appendix 10 demonstrates that the first period policy maker will achieve the following pattern of taxation over time,

$$\tau_0 = \frac{1}{2\nu_1 - 1}\tau_1 + \frac{\beta(1-\nu_1)}{2\nu_1 - 1}\tau_2$$
$$\tau_0 = \frac{(1-\nu_1)}{2\nu_1 - 1}\tau_1 + \frac{1}{2\nu_2 - 1}\tau_2$$

which can be equated to yield,

$$\tau_2 = (2\nu_1 - 1)\tau_1 + (1-\nu_1)(1+\beta(2\nu_2 - 1))\nu_1^{-1}\tau_2 \tag{31}$$

Despite an inability to commit, the period $t = 0$ policy maker can achieve this desired evolution of policy by issuing an appropriate amount of two-period debt such that ICC (29) is isomorphic to this expression. How debt maturity is used by the period $t = 0$ policy maker can be seen by contrasting (31) with what would be chosen by the period $t = 1$ policy maker in an environment with only single period debt, $\tau_2 = \tau_1(2\nu_2 - 1)$. This implies that the first policy maker wishes the period $t = 1$ policy maker to reduce debt by less, delaying some of the fiscal adjustment to period $t = 2$. By issuing two-period bonds they, therefore, reduce the debt-stabilization bias in period $t = 1$, levy less taxation and, likely, mitigate the inflation bias too - see Eq. (28).

The complete equilibrium is shown in Fig. B.3 which contrasts outcomes under commitment and discretion (as well as the case of time-consistent policy with only short-term debt) as a function of θ. For all values of θ considered, an inability to commit means that the reduction in debt is front loaded in period $t = 0$ as a result of the debt-stabilization bias. As described above, with only single period debt, debt continues to be stabilized aggressively with tax rates falling over time. In contrast the ability to issue two period debt reduces the inflationary and debt-stabilization biases in period $t = 1$, allowing the policy maker to reduce taxes in that period without being adversely impacted by higher inflation. This slowing of debt stabilization in period $t = 1$ then results in higher taxes and inflation in the final period. As the costs of inflation increase, the desire to lengthen debt maturity to slow the pace of debt reduction in period $t = 1$ is reduced and the stock of short-term debt switches from negative to positive, thereby reducing overall debt maturity.

5.4.2. Full model

In order to assess whether or not the benchmark model exhibits the same properties we augment it to include single period debt alongside the longer-maturity debt enabling the policy maker to adjust the average maturity of the debt stock. The wealth of the existing bondholders entering period t is now $D_t \equiv (1+\rho P_t^M)B_{t-1}^M + B_{t-1}^S$, the household then buys bonds, $P_t^M B_t^M + P_t^S B_t^S$ and as a result the government's budget constraint becomes,

$$P_t^M b_t^M + P_t^S b_t^S = \frac{b_{t-1}^S}{\Pi_t} + (1+\rho P_t^M)\frac{b_{t-1}^M}{\Pi_t} - \frac{W_t}{P_t}N_t\tau_t + G_t + tr$$

The remainder of the policy problem is unchanged, except for the fact that policy functions now have three arguments, the elasticity of substitution between goods, ϵ_t, and the levels of both maturities of bond, b_{t-1}^S and b_{t-1}^M. Online Appendix 11 derives the resultant FOCs. The implication of the analysis above is that in the extended benchmark model any parameter change which reduces the inflationary bias problem is likely to result in an increase in the proportion of long-term debt if the policy maker is given the opportunity to issue both short and long-term debt. Therefore, we conjecture that reduced price stickiness and markups and greater myopia should all lead to a greater reliance on long-term debt, and may even result in the policy maker accumulating a short-term asset in order to leverage the benefits of issuing long-term debt.

Table A.4 contains steady-state debt-to-GDP ratios for the benchmark model alongside variants which allow the policy maker to simultaneously issue short-term debt (possibly in negative quantities). Comparing the first two columns it can be seen that for the benchmark calibration the costs of inflation are sufficiently high that the policy chooses to shorten maturity in line with the results for high values of θ above. Instead if we reduce the degree of price stickiness in column three, the desire to leverage long-term debt as a means of reducing the debt stabilization bias becomes apparent and the government accumulates short-term assets in order to raise the proportion of longer-term debt. The next column reduces the mark-up which, by reducing the inflationary bias problem, ceteris paribus, also allows the policy maker to lengthen maturity by reducing the proportion of short-term debt in overall debt. Finally, increasing the myopia of the policy maker reduces the debt-stabilization bias and encourages the policy maker to issue more long-term debt.

The use of debt maturity in this way also occurs in response to shocks. Fig. B.4 considers the response to a markup shock when the policy maker can issue short-term debt. The increase in the inflation bias caused by the rise in the markup reduces the current policy maker's desire to delay future debt reduction and so they issue relatively more short-term debt. Outside of the initial period, this causes the policy maker to moderate the rise in taxation (which is otherwise inflationary) resulting in a medium term increase in the debt-to-GDP ratio in contrast to the benchmark model. In the first period the policy maker undertakes a sharp tightening of monetary policy which induces a fall in bond prices, making it cheaper for the government to retire those bonds. The fiscal consequences of this are offset by an associated rise in taxation in the initial period.

6. Conclusions

The existence of nominal debt induces a state-dependent inflation bias problem as the policy maker wishes to utilize inflation surprises to offset monopolistic competition and tax distortions and reduce the real value of debt. This temptation is greater with higher debt levels and shorter debt maturity, resulting in a debt stabilization bias as the policy maker deviates from Ramsey policy by returning debt to steady-state to mitigate the associated inflation biases.

The response to shocks in such an environment seeks to avoid exacerbating these biases, and is radically different from policy under commitment as a result. Endogenizing the debt maturity decision gives the current policy maker an additional tool through which to influence the pace of future debt stabilization - lengthening debt maturity when the underlying costs associated with the inflation bias are reduced and vice versa.

The dependence of the inflationary bias on both the level and maturity of government debt highlighted by the paper, implies an obvious area for future research would be to explore how monetary policy institutions can be insulated from such effects, given the apparent inability of central banks to commit (see Chen et al., 2017) means that even an independent monetary authority has entered into a strategic game with the fiscal policy maker.

Appendix A. Tables

Table A.1
Parameterization.

Parameter	Value	Definition
β	0.995	Quarterly discount factor, household.
$\widetilde{\beta}$	0.982	Quarterly discount factor, policy maker.
σ	2	Relative risk aversion coefficient
σ^g	2	Relative risk aversion coefficient for government spending
φ	3	Inverse Frish elasticity of labor supply
$\overline{\epsilon}$	14.33	Elasticity of substitution between varieties
ρ	0.95	Debt maturity structure (5 years)
χ	0.0076	Scaling parameter associated with government spending
ρ_ϵ	0.939	AR-coefficient of cost-push shock
σ_ϵ	0.052	Standard deviation of cost-push shock
ϕ	50	Rotemberg adjustment cost coefficient

Table A.2
Steady-state: myopia, price flexibility and monopolistic competition.

Variable	Data	Benchmark	No myopia	Myopia $\widetilde{\beta}=0.975$	Price flexibility $\phi=30$	Markup $\frac{\overline{\epsilon}}{\overline{\epsilon}-1}=5\%$
$\frac{bP^M}{4Y}$	31.2%	31.2%	−152.9%	75.6%	36.7%	89.8%
(Π^4-1)	3.5%/2.4%[a]	3.0%	−1.1%	4.5%	3.8%	3.7%
(R^4-1)	5.66%/4.9%	5.1%	0.9%	6.7%	5.9%	5.8%
Y	N.A.	0.977	0.985	0.975	0.977	0.980
G/Y	7.84%	7.82%	7.93%	7.76%	7.81%	7.75%
τ	17.5%	18.9%	15.3%	19.8%	19.0%	19.5%

[a] Over the full sample the average inflation rate was 3.5% (with a standard deviation of 2.3%), while following the Great Moderation (post 1985) the average inflation rate falls to 2.4% with a standard deviation of 0.76%.

Table A.3
Steady-state: maturity.

Variable	Benchmark	1 Qtr Maturity $\rho=0$	1 Yr Maturity $\rho=0.7538$	10 Yr Maturity $\rho=0.9799$	30 Yr Maturity $\rho=0.9966$
$\frac{bP^M}{4Y}$	31.2%	−11.1%	12.8%	53.6%	102.0%
(Π^4-1)	3.0%	1.5%	2.5%	3.62%	5.1%
(R^4-1)	5.1%	3.5%	4.6%	5.7%	7.2%
Y	0.977	0.979	0.978	0.976	0.973
G/Y	7.82%	8.01%	7.80%	7.83%	7.82%
τ	18.9%	18.2%	18.4%	19.4%	20.4%

Table A.4
Steady-state debt-to-GDP ratios: myopia, price flexibility and monopolistic competition.

	Benchmark	Benchmark Endog. Maturity	Price flexibility $\phi=10$	Lower Markup $\frac{\overline{\epsilon}}{\overline{\epsilon}-1}=5\%$	Myopia $\widetilde{\beta}=0.975$
Debt to GDP Ratio(%)	31.2%	31.2%	88.2%	24.7%	74.7%
Share of Single Period Debt(%)	0	4.4%	0.9%	−0.7%	2.4%

Appendix B. Figures

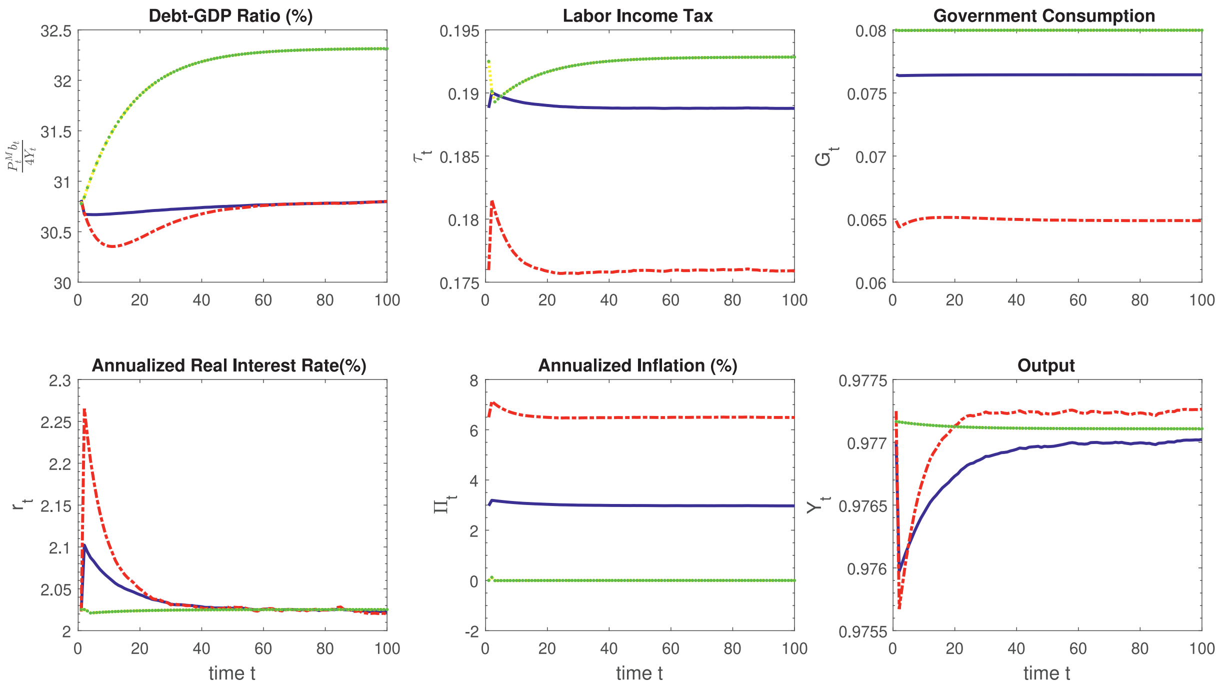

Fig. B.1. Markup shock - commitment vs. discretion - debt maturity. *Note:* Yellow dotted line represents outcomes under commitment with long-term debt, and green points commitment with single period debt. These largely overlap. Solid blue line presents discretion with long-term debt, and red dash-dotted line discretion with single period debt. Myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the other model variants considered. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

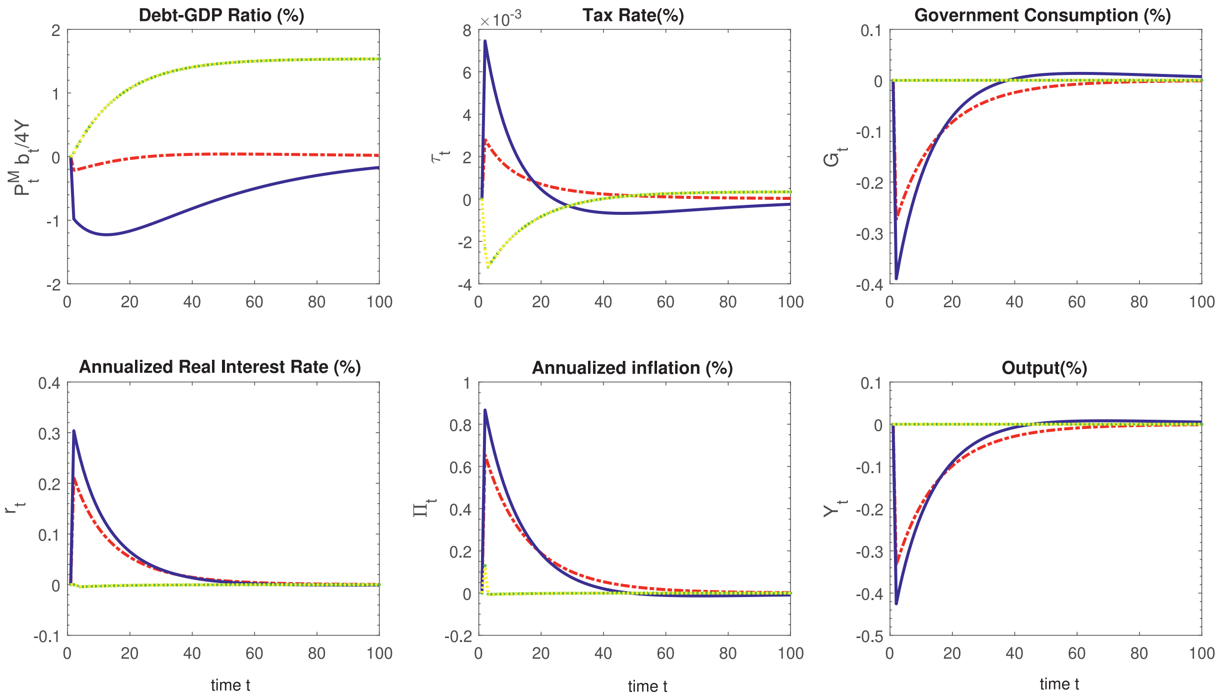

Fig. B.2. Markup shock - commitment vs. discretion - level of debt. *Note:* Yellow dotted line represents outcomes under commitment with high (52% of GDP) levels of debt, and green points commitment with low (16% of GDP) levels of debt. These largely overlap. Solid blue line presents discretion with high levels of steady-state debt, and red dash-dotted line discretion with low levels of steady-state debt. Debt, taxes, interest rates and inflation are expressed in deviations from stochastic steady-state. Government consumption and output in percentage deviations from stochastic steady-state. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

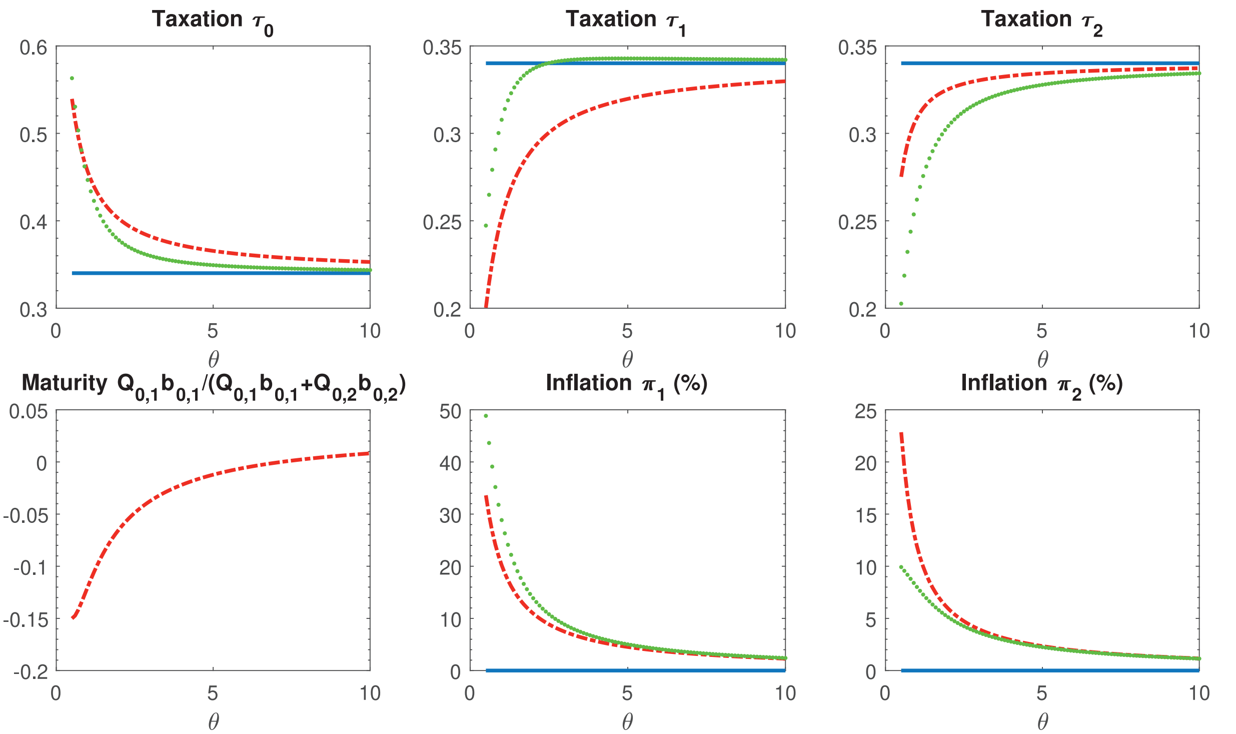

Fig. B.3. Equilibrium outcomes for the three-period model. *Note:* The figure gives the equilibrium outcomes for the three-period model as a function of the cost of inflation, θ. Three cases are considered - commitment (solid blue line), discretion with single period debt only (green dots) and discretion with one and two-period bonds (red dot-dashed line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

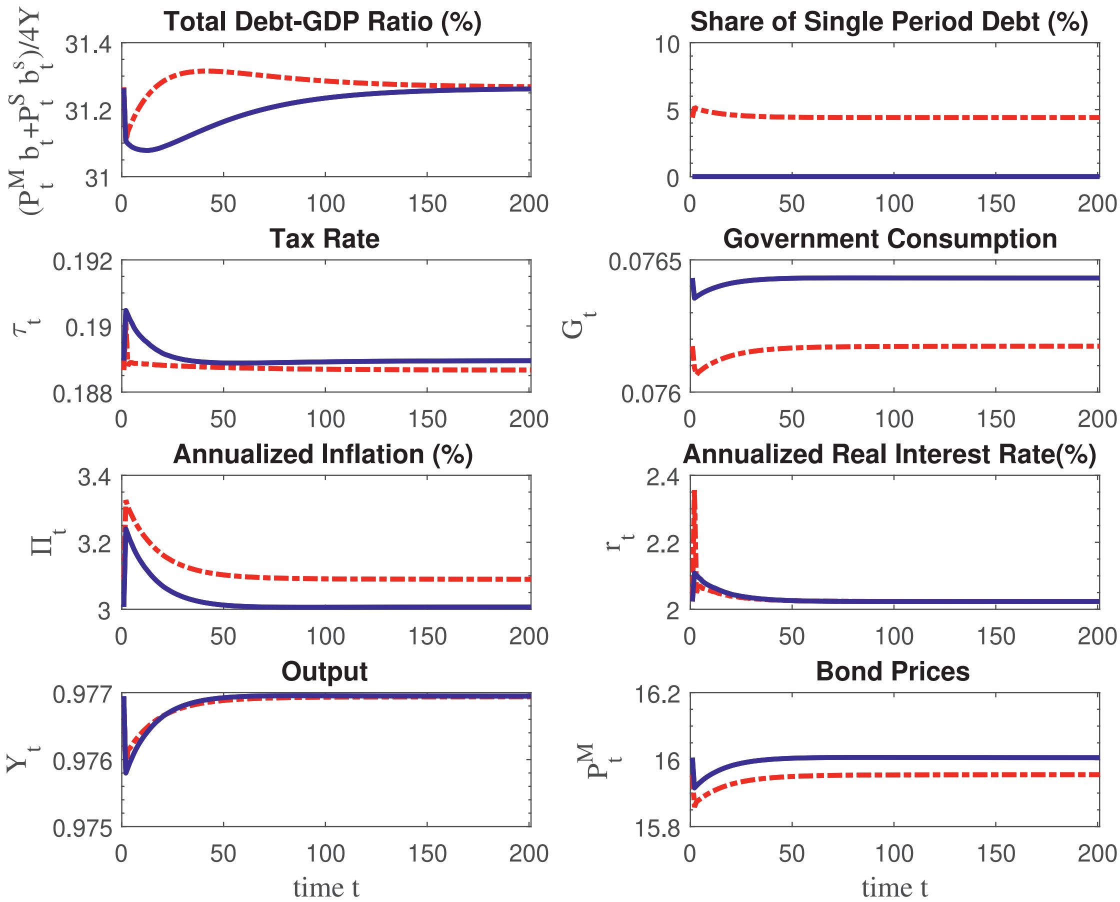

Fig. B.4. Endogenous debt maturity and fiscal consolidation. *Note:* Endogenous debt maturity - red dash-dotted line. Benchmark case of exogenous debt maturity - solid blue line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2020.03.015.

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